Clarendon Lectures in Finance: The Adaptive Markets Hypothesis

Andrew W. Lo, MIT

Lecture 1: Evolutionary Foundations of Behaviour and Rationality

June 12, 2013
Expected Utility Theory

\[
\max_{C} E[U(C)] \quad \text{s.t.} \quad C \in B
\]

\[
U'(\phi) > 0 \quad ; \quad U''(\phi) < 0
\]
Samuelson (1998) on the Origins of *Foundations*

Perhaps most relevant of all for the genesis of *Foundations*, Edwin Bidwell Wilson (1879–1964) was at Harvard. Wilson was the great Willard Gibbs's last (and, essentially only) protege at Yale. He was a mathematician, a mathematical physicist, a mathematical statistician, a mathematical economist, a polymath who had done first-class work in many fields of the natural and social sciences. I was perhaps his only disciple... I was vaccinated early to understand that economics and physics could share the same formal mathematical theorems (Euler's theorem on homogeneous functions, Weierstrass's theorems on constrained maxima, Jacobi determinant identities underlying Le Chatelier reactions, etc.), while still not resting on the same empirical foundations and certainties.
Theory of Economic Behavior

Physics Approach In Economics Led To:

- Utility theory, revealed preference (Samuelson)
- General equilibrium theory (Arrow, Debreu)
- Game theory (Harsanyi, Nash, Selten, Shapley)
- Macroeconometrics (Klein, Tinbergen)
- Portfolio theory (Markowitz, Tobin)
- Rational expectations (Lucas, Muth, Sargent)
- Option-pricing theory (Black, Merton, Scholes)
- Efficient markets (Fama, Samuelson)
Theory of Economic Behavior

Theory of Market Efficiency:

- Samuelson (1965), Fama (1965, 1970)
- No Free Lunch, No Arbitrage
- Prices Fully Reflect All Available Information
- Prices Follow Random Walks
- Technical Analysis Is Futile
- Trade-Off Between Risk and Expected Return
- “Active” Management Does Not Add Value

“The Invisible Hand”, “The Wisdom of Crowds”
Behavioral Critique

- Rationality is not supported by the data
- Cognitive and behavioral biases
  - Loss aversion, anchoring, framing
  - Probability matching
  - Overconfidence
  - Overreaction
  - Herding
  - Mental accounting
  - etc.

© 2013 by Andrew W. Lo
All Rights Reserved
Consider Repeated Coin-Toss Guessing Game:

- If you’re correct, you get $1, otherwise –$1
- Suppose coin is biased (75% H, 25% T)
- Profit-maximizing strategy:
- Actual behavior: HHHHTHHHHTHHHHHTHHHHTH
- Common to ants, fish, pigeons, primates, etc.
- Why?
- Is it irrational or **adaptive**?
Reconciling Efficient Markets with Behavioral Finance

- Behavioral biases are adaptive behaviors taken out of their natural context
- Behaviors may be suboptimal from an individual’s perspective, but growth-optimal from the population perspective, hence they can persist
- Variation and natural selection shapes behavior
- The key is how behavior and the environment interact to determine “reproductive success”
- Evolution applied to financial interactions
Overview of Adaptive Markets

Lecture 1. The Evolutionary Origin of Behavior

- Formal mathematical model of the evolutionary origin of behavior
- Key idea is how behavior interacts with the type of risk affecting reproductive success
- Systematic risk $\Rightarrow$ seemingly suboptimal behavior
- Idiosyncratic risk $\Rightarrow$ seemingly optimal behavior
- This can explain both rationality and its departures, yields a natural definition of intelligence, and provides a formal theory of bounded rationality

- Efficient markets is the “frictionless ideal”, but reality contains many frictions which are important (Coase)
- New insights from the cognitive neurosciences regarding the meaning of rationality and the neural components of behavior
- An evolutionary interpretation of bounded rationality and intelligence
- Efficient markets and behavioral departures are part of the same ecology
Overview of Adaptive Markets

Lecture 3. Hedge Funds: The Galapagos Islands of Finance

- Evolution can be easily observed in the hedge fund industry because of its speed of innovation; behavioral patterns and arbitrage activity shape market dynamics
- Hedge funds may be used as early warning indicators of financial distress and systemic risk
- The evolutionary perspective changes everything!
Literature Review

- Behavioral economics and finance
  - Thaler, Shefrin, Statman, Shiller, Shleifer, Laibson
- Psychology and cognitive sciences
  - Simon, Tversky, Kahneman, Pinker, Prelec, Tenenbaum
- Evolutionary psychology and sociobiology
  - Wilson, Hamilton, Trivers, Cosmides, Tooby, Gigerenzer
- Evolutionary game theory and economics
  - Malthus, Schumpeter, von Hayek, Maynard Smith, Nowak, Robson, L. Samuelson
- Behavioral ecology and evolutionary biology
  - Darwin, Levin, Clarke
Binary Choice Model

Consider “Asexual Semelparous” Individuals

- Individual lives one period, makes one decision, \( a \) or \( b \)
- Generates offspring \( x = x_a \) or \( x_b \), then dies
- Offspring behaves exactly like parent

\[
\begin{align*}
x_i^f &= I_i^f x_a + (1 - I_i^f) x_b \\
I_i^f &\equiv \begin{cases} 
1 & \text{with probability } f \\
0 & \text{with probability } 1-f 
\end{cases} \\
f &\in [0, 1]
\end{align*}
\]
Binary Choice Model

Consider “Asexual Semelparous” Individuals

- If $f = 1$, individual always chooses $a$ (offspring too)
- If $f = 0$, individual always chooses $b$ (offspring too)
- If $0 < f < 1$, individual randomizes with prob. $f$ and offspring also randomizes with same $f$

Impact of behavior on reproductive success: $\Phi(x_a, x_b)$

- Summarizes environment and behavioral impact on fitness
- Links behavior directly to reproductive success
- Contains all genetic/environmental interactions
- Biological/behavioral “reduced form”
Consider “Asexual Semelparous” Individuals

- This is repeated over many generations
Binary Choice Model

Consider “Asexual Semelparous” Individuals

- Initial population is uniformly distributed on [0,1]

Which behavior dominates?
Consider “Asexual Semelparous” Individuals

(A1) Assume: $\Phi(x_a, x_b)$ is \textbf{identical} across individuals

- $\Phi(x_a, x_b)$ is \textbf{IID} across time; $I_i^f$ also \textbf{IID}

- Individuals are “mindless”, \textbf{not} strategic optimizers
- Which $f$ survives over many generations?
- In other words, what kind of behavior evolves?
- Evolution is the “process of elimination” (E. Mayr)
- Mathematics: find the $f^*$ that maximizes growth rate
- This $f^*$ will be the behavior that \textit{survives} and \textit{flourishes}
\[ n_t^f = \sum_{i=1}^{n_{t-1}^f} x_{i,t}^f = \left( \sum_{i=1}^{n_{t-1}^f} I_{i,t}^f \right) x_{a,t} + \left( \sum_{i=1}^{n_{t-1}^f} (1 - I_{i,t}^f) \right) x_{b,t} \]
Which type of individuals will grow fastest?

The $f^*$ that maximizes $\mu(f)$ on [0,1]:

$$f^* \equiv \arg\max_f \mu(f) = \arg\max_f \mathbb{E}[\log(f x_a + (1-f)x_b)]$$

$$\mu''(f) = -\mathbb{E} \left[ \frac{(x_a - x_b)^2}{\log^2(f x_a + (1-f)x_b)} \right] < 0$$

$\mu(f)$ is strictly concave on [0,1]; unique maximum

Three possibilities: $1(f)$

- $f^* = 0$
- $f^* = 1$

---

Lecture 1

© 2013 by Andrew W. Lo
All Rights Reserved
Population Arithmetic

- Growth-optimal $f^*$ given by:

$$f^* = \begin{cases} 
1 & \text{if } E[x_a/x_b] > 1 \text{ and } E[x_b/x_a] < 1 \\
\text{solution to (1)} & \text{if } E[x_a/x_b] \geq 1 \text{ and } E[x_b/x_a] \geq 1 \\
0 & \text{if } E[x_a/x_b] < 1 \text{ and } E[x_b/x_a] > 1 
\end{cases}$$

$$0 = E \left[ \frac{x_a - x_b}{f^*x_a + (1 - f^*)x_b} \right] \quad (1)$$

$$E \left[ \frac{x_a}{f^*x_a + (1 - f^*)x_b} \right] = E \left[ \frac{x_b}{f^*x_a + (1 - f^*)x_b} \right]$$
Population Arithmetic

How Does $f^*$ Persist? By Natural Selection:

$$\left( \frac{n_T^{f'}}{n_T^{f^*}} \right)^{1/T} \xrightarrow{p} \exp\left( [\mu(f') - \mu(f^*)] \right) \xrightarrow{p} 0$$

$$\Rightarrow \frac{n_T^{f'}}{n_T^{f^*}} \xrightarrow{p} 0$$

- $f^*$ type takes over exponentially fast
- Behavior $f^*$ is optimal for the population
- Behavior $f^*$ is not necessarily optimal for the individual
- This requires no intention, deliberation, or intelligence
- Contrast this behavior with utility maximization!
Consider Special Case For \( \Phi(x_a, x_b) \):

<table>
<thead>
<tr>
<th>Action</th>
<th>State 1 (prob. ( p ))</th>
<th>State 2 (prob. ( 1-p ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( x_a = m )</td>
<td>( x_a = 0 )</td>
</tr>
<tr>
<td>( b )</td>
<td>( x_b = 0 )</td>
<td>( x_b = m )</td>
</tr>
</tbody>
</table>

- Outcomes \( x_a \) and \( x_b \) are perfectly out of phase
  
  \[
  \mu(f) = \log m + p \log f + (1 - p) \log(1 - f) 
  \]
  
  \[
  f^* = p 
  \]

- Probability matching!

- This behavior will dominate the population (eventually)
Probability Matching Explained

Consider A Simple Ecology with Rain/Shine:

- Decision: build nest in a or b?
- Optimize or randomize?

\[ f \quad \text{(} p = 0.75 \text{)} \quad (1 - p = 0.25) \]

\[ \begin{align*}
\mathbf{a} : & \quad x_a = 3 \\
\mathbf{b} : & \quad x_b = 0
\end{align*} \]
### Probability Matching Explained

<table>
<thead>
<tr>
<th>Generation</th>
<th>( f = 0.20 )</th>
<th>( f = 0.50 )</th>
<th>( f^* = 0.75 )</th>
<th>( f = 0.90 )</th>
<th>( f = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>6</td>
<td>12</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>6</td>
<td>6</td>
<td>57</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>12</td>
<td>12</td>
<td>144</td>
<td>270</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>9</td>
<td>24</td>
<td>387</td>
<td>810</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>18</td>
<td>48</td>
<td>1,020</td>
<td>2,430</td>
</tr>
<tr>
<td>6</td>
<td>96</td>
<td>21</td>
<td>108</td>
<td>2,766</td>
<td>7,290</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>42</td>
<td>240</td>
<td>834</td>
<td>21,870</td>
</tr>
<tr>
<td>8</td>
<td>45</td>
<td>54</td>
<td>528</td>
<td>2,292</td>
<td>65,610</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>87</td>
<td>1,233</td>
<td>690</td>
<td>196,830</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>138</td>
<td>2,712</td>
<td>204</td>
<td>590,490</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>204</td>
<td>6,123</td>
<td>555</td>
<td>1,771,470</td>
</tr>
<tr>
<td>12</td>
<td>36</td>
<td>294</td>
<td>13,824</td>
<td>159</td>
<td>5,314,410</td>
</tr>
<tr>
<td>13</td>
<td>87</td>
<td>462</td>
<td>31,149</td>
<td>435</td>
<td>15,943,230</td>
</tr>
<tr>
<td>14</td>
<td>42</td>
<td>768</td>
<td>69,954</td>
<td>1,155</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>27</td>
<td>1,161</td>
<td>157,122</td>
<td>3,114</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>15</td>
<td>1,668</td>
<td>353,712</td>
<td>8,448</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>2,451</td>
<td>795,171</td>
<td>22,860</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>3,648</td>
<td>1,787,613</td>
<td>61,734</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>9</td>
<td>5,469</td>
<td>4,020,045</td>
<td>166,878</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>8,022</td>
<td>9,047,583</td>
<td>450,672</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>6</td>
<td>12,213</td>
<td>6,786,657</td>
<td>1,215,723</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>18,306</td>
<td>15,272,328</td>
<td>366,051</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>0</td>
<td>27,429</td>
<td>34,366,023</td>
<td>987,813</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>41,019</td>
<td>77,323,623</td>
<td>2,667,984</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>61,131</td>
<td>173,996,290</td>
<td>7,203,495</td>
<td>0</td>
</tr>
</tbody>
</table>

\( p = 0.75 \)

\( m = 3 \)
Probability Matching Explained

What About the “Optimal” Strategy for the Individual?

- Suppose $p > \frac{1}{2}$; then $\hat{f} = 1$
- The first time $x_a = 0$, all individuals of this type vanish
- This behavior cannot persist; $f^*$ persists
- $f^*$ may be interpreted as a primitive version of altruism

When Is Probability Matching Advantageous?

- When two choices are highly negatively correlated
- Diversification improves likelihood of survival
- “Nature abhors an undiversified bet”
Probability Matching Explained

Now Consider a More General \( \Phi(x_a, x_b) \)

\[
\begin{align*}
\text{Prob}(x_a = c_{a1}, x_b = c_{b1}) &= p \in [0, 1] \\
\text{Prob}(x_a = c_{a2}, x_b = c_{b2}) &= 1 - p \equiv q \\
0 &\leq c_{ij}, \quad i = a, b, \quad j = 1, 2 \\
0 &\neq c_{aj} + c_{bj}
\end{align*}
\]

- Then the growth-optimal behavior \( f^* \) depends only on

\[
\begin{align*}
   r_j &\equiv \frac{c_{aj}}{c_{bj}}, \quad j = 1, 2
\end{align*}
\]
Probability Matching Explained

- Growth-optimal behavior $f^*$ given by:

$$f^* = \begin{cases} 
1 & \text{if } r_2 \in [q + \frac{pq}{r_1-p}, \infty) \text{ and } r_1 > p \\
\frac{p}{1-r_2} + \frac{q}{1-r_1} & \text{if } \begin{cases} 
  r_2 \in \left(\frac{1}{q} - \frac{p}{q}r_1, q + \frac{pq}{r_1-p}\right) \text{ and } r_1 > p, \text{ or } \\
  r_2 \in \left(\frac{1}{q} - \frac{p}{q}r_1, \infty\right) \text{ and } r_1 \leq p 
\end{cases} \\
0 & \text{if } r_2 \in [0, \frac{1}{q} - \frac{p}{q}r_1]
\end{cases}$$

$$f^* = p \left( 1 + \mathcal{O}(1/r_1) + \mathcal{O}(r_2) \right)$$

$$\approx p \quad \text{if } r_1 \gg 0, \quad r_2 \ll 1$$
Probability Matching Explained

Exact probability matching condition:

\[ 0 = p \frac{r_2}{1 - r_2} + q \frac{1}{1 - r_1} \]
Risk Preferences

To Study Risk Preferences, Let $b$ Be “Riskless”

\[
\begin{align*}
\text{Prob}(x_a = c_{a1}, x_b = c_b) &= p \in [0, 1] \\
\text{Prob}(x_a = c_{a2}, x_b = c_b) &= 1 - p \equiv q \\
\text{and} & \quad c_{a1} < c_b < c_{a2}
\end{align*}
\]

- Then growth-optimal behavior is given by:

\[
f^* = \begin{cases} 
1 & \text{if } c_b \in [c_{a1}, \hat{c}_a) \\
\frac{(\hat{c}_a - c_b)c_b}{(c_{a2} - c_b)(c_b - c_{a1})} & \text{if } c_b \in [\hat{c}_a, \overline{c}_a) \\
0 & \text{if } c_b \in [\overline{c}_a, c_{a2}] 
\end{cases}
\]

\[
\hat{c}_a \equiv \frac{1}{p/c_{a1} + q/c_{a2}} \\
\overline{c}_a \equiv pc_{a1} + qc_{a2}
\]
Risk Preferences

- Growth-optimal behavior $f^*$:

  - Always choose risky option $a$ if $f^* = 1$
  - Randomize choice with probability $f^*$
  - Always choose safe option $b$ if $f^* = 0$

- Now suppose $c_b = \overline{c}_a$ (actuarially fair gamble)
  - Which is “preferred”, i.e., what is $f^*$?
Risk Preferences

- Growth-optimal behavior is $f^* = 0$:
- Risk aversion is evolutionarily dominant!
- Follows from Jensen’s Inequality: $3 \times 3 > 2 \times 4$

- For $s \equiv c_{a2}/c_{a1} \approx 1$, choice is deterministic
- When $s \gg 1$, randomization is more common
Risk Aversion

- Define risky outcomes relative to riskless outcome

\[ c_{a1} = c_b - d \]

\[ c_{a2} = c_b + u, \quad u, d > 0 \]

- Suppose \( p = \frac{1}{2} \) and \( f^* = \frac{1}{2} \) (indifferent between \( a \) and \( b \))

\[ u = d + \frac{d^2}{c_b - d} > d \quad \text{(greater upside required)} \]

\[ \pi = u - d = \frac{(c_b - c_{a1})^2}{c_{a1}} \quad \text{(evolutionary risk premium)} \]

- Risk premium is homogeneous of degree 1

- No equilibrium model needed!
Systematic vs. Idiosyncratic Risk

With “Systematic” Risk, Natural Selection Can Explain:

- Probability matching
- Randomization
- Risk aversion and risk-sensitive foraging behavior
- Loss aversion, anchoring, framing

What If $\Phi(x_a, x_b)$ Is Not Identical Across Individuals?

(A1') Assume: $\Phi(x_a, x_b)$ is IID across individuals

- $\Phi(x_a, x_b)$ is IID across time; $I_{it}^f$ also IID

- **Idiosyncratic** reproductive risk
Consider “Asexual Semelparous” Individuals

\[ x_i^f = I_i^f x_{a,i} + (1 - I_i^f) x_{b,i}, \quad I_i^f = \begin{cases} 1 & \text{with probability } f \\ 0 & \text{with probability } 1 - f \end{cases} \]

- Individuals are “mindless”, not strategic optimizers
- Which \( f \) survives over many generations?
- In other words, what kind of behavior evolves?
- Evolution is the “process of elimination” (E. Mayr)
- Mathematics: find the \( f \) that maximizes growth rate
- This \( f^* \) will be the behavior that survives and flourishes
Systematic vs. Idiosyncratic Risk

\[ n^f_t = \sum_{i=1}^{n^f_{t-1}} x^f_{i,t} \neq \left( \sum_{i=1}^{n^f_{t-1}} I^f_{i,t} x_{a,t} \right) + \left( \sum_{i=1}^{n^f_{t-1}} (1 - I^f_{i,t}) x_{b,t} \right) \]

\[ n^f_t = \sum_{i=1}^{n^f_{t-1}} x^f_{i,t} = \left( \sum_{i=1}^{n^f_{t-1}} I^f_{i,t} x_{a,i,t} \right) + \left( \sum_{i=1}^{n^f_{t-1}} (1 - I^f_{i,t}) x_{b,i,t} \right) \]

\[ n^f_t \overset{p}{=} n^f_{t-1} (f \mu_a + (1-f) \mu_b) \]

\[ n^f_T \overset{p}{=} \prod_{t=1}^{T} (f \mu_a + (1-f) \mu_b) = \exp \left( \sum_{t=1}^{T} \log (f \mu_a + (1-f) \mu_b) \right) \]

\[ \frac{1}{T} \log n^f_T \overset{p}{=} \frac{1}{T} \sum_{t=1}^{T} \log (f \mu_a + (1-f) \mu_b) \]

\[ \mu(f) \equiv E[\log (f \mu_a + (1-f) \mu_b)] \]

Non-stochastic!
Systematic vs. Idiosyncratic Risk

Let The Environment Be Rain/Shine Microclimates:

- Decision: build nest in a or b?
- Optimize or randomize?

\[ f \]  
\[ 1-f \]

\[ x_a = 3 \] \[ x_a = 0 \]
\[ x_b = 0 \] \[ x_b = 3 \]

\( p = 0.75 \) \[ (1 - p = 0.25) \]
# Systematic vs. Idiosyncratic Risk

<table>
<thead>
<tr>
<th>Generation</th>
<th>$f = 0.20$</th>
<th>$f = 0.50$</th>
<th>$f^* = 0.75$</th>
<th>$f = 0.90$</th>
<th>$f = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>15</td>
<td>42</td>
<td>72</td>
<td>54</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>27</td>
<td>87</td>
<td>177</td>
<td>120</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>45</td>
<td>168</td>
<td>357</td>
<td>270</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>60</td>
<td>300</td>
<td>717</td>
<td>588</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>84</td>
<td>591</td>
<td>1,488</td>
<td>1,329</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>141</td>
<td>1,074</td>
<td>3,174</td>
<td>2,955</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>207</td>
<td>2,007</td>
<td>6,669</td>
<td>6,555</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>315</td>
<td>3,759</td>
<td>14,241</td>
<td>14,748</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>492</td>
<td>7,152</td>
<td>29,733</td>
<td>33,060</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>705</td>
<td>13,398</td>
<td>62,214</td>
<td>74,559</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1,053</td>
<td>25,071</td>
<td>130,317</td>
<td>167,703</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>1,635</td>
<td>46,623</td>
<td>273,834</td>
<td>377,037</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>2,427</td>
<td>87,333</td>
<td>575,001</td>
<td>849,051</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>3,663</td>
<td>163,092</td>
<td>1,206,849</td>
<td>1,910,031</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>5,433</td>
<td>305,091</td>
<td>2,536,023</td>
<td>4,296,213</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>8,148</td>
<td>570,852</td>
<td>5,325,852</td>
<td>9,666,762</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>12,264</td>
<td>1,069,884</td>
<td>11,188,509</td>
<td>21,755,844</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>18,453</td>
<td>2,007,642</td>
<td>23,494,611</td>
<td>48,959,286</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>27,711</td>
<td>3,763,281</td>
<td>49,346,967</td>
<td>110,148,060</td>
</tr>
</tbody>
</table>

$p = 0.75$

$m = 3$
Growth-Optimal Behavior With Idiosyncratic Risk:

\[ f^* = \begin{cases} 
1 & \text{if } \mu_a > \mu_b \\
0 & \text{if } \mu_a \leq \mu_b 
\end{cases} \]

- In this case, no difference between individually optimal and growth-optimal behavior
- No “behavioral biases”; no risk aversion; everyone behaves “rationally” (*Homo economicus*)
- Behavior can be identical because environment is not
- If environment is identical, behavior cannot be
- “Nature abhors an undiversified bet”!
Extensions of the Binary Choice Model

- Sexual reproduction (mutation), iteroparity (binomial tree), multinomial choice
- Multivariate multi-stage choice problems (simulation)
- Other assumptions for $\Phi(x_a, x_b)$ can generate well-known results in economics and evolutionary biology
  - Regime-switching yields “punctuated equilibria”
  - Common factors yield “group selection”
  - Resource constraints yield “strategic” behavior
  - Memory and conditioning information yield “forward-looking” and “planning” behavior, i.e., an evolutionary definition of intelligence
Conclusion

- Framework for modeling the evolution of behavior
  - Abstracts from underlying genetics
  - Biological “reduced form” model of behavior
- Simplicity implies behaviors are primitive and ancient
- Mathematical basis of the Adaptive Markets Hypothesis
  - Evolution determines individual behavior
  - Evolution also determines aggregate dynamics
  - Efficiency and irrationality are both adaptive
  - The key is how environment is related to behavior to yield bounded rationality and intelligence
Thank You!
Further Reading