Legal-system Arbitrage and Parent-Subsidiary Capital Structures

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Legal-system Arbitrage and Parent–Subsidiary Capital Structures

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March 15, 2012

Abstract

This paper develops a new theory of the capital structure of parent–subsidiary organization based on legal-system arbitrage: The optimal capital structure for the parent–subsidiary organization minimizes the value of the ex post opportunism options created by the organization’s ability to selectively renegotiate claims of the component legal entities. This theory explains the complex mix between parent and subsidiary debt financing observed in most parent–subsidiary organizations, particularly multinational corporations and conglomerates, even in the absence of tax and private information effects. Optimal capital structures minimize the default premia associated with the organization’s overall financing package by equating the marginal enforceability of debt contracts across subsidiaries. Consistent with empirical findings in Kolasinski (2009), we show that significant borrowing at the subsidiary level is optimal. Consistent with empirical findings in Desai et al. (2004), we show that the parent corporation’s utilization of subsidiary debt financing is positively related to the creditor-friendliness of the legal regime in which the parent company operates. Further, our model shows firms operating across legal boundaries gain from obtaining some financing even in the locations featuring less creditor-friendly legal regimes, though they will aim to structure debt ex ante to commit to ex post renegotiation under the most efficient regimes. In addition, our model produces many new empirical predictions regarding issues such as the optimal allocation of capital and overall financing policy.

JEL Classification Codes: G15, G32, G33, G38, K4

Keywords: Conglomerate, Capital structure, Debt renegotiation, Multinational corporation, Parent-Subsidiary organization, Subsidiary financing

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Abstract

This paper develops a new theory of the capital structure of parent–subsidiary organization based on legal-system arbitrage: The optimal capital structure for the parent–subsidiary organization minimizes the value of the ex post opportunism options created by the organization’s ability to selectively renegotiate claims of the component legal entities. This theory explains the complex mix between parent and subsidiary debt financing observed in most parent–subsidiary organizations, particularly multinational corporations and conglomerates, even in the absence of tax and private information effects. Optimal capital structures minimize the default premia associated with the organization’s overall financing package by equating the marginal enforceability of debt contracts across subsidiaries. Consistent with empirical findings in Kolasinski (2009), we show that significant borrowing at the subsidiary level is optimal. Consistent with empirical findings in Desai et al. (2004), we show that the parent corporation’s utilization of subsidiary debt financing is positively related to the creditor-friendliness of the legal regime in which the parent company operates. Further, our model shows firms operating across legal boundaries gain from obtaining some financing even in the locations featuring less creditor-friendly legal regimes, though they will aim to structure debt ex ante to commit to ex post renegotiation under the most efficient regimes. In addition, our model produces many new empirical predictions regarding issues such as the optimal allocation of capital and overall financing policy.

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1 Introduction

Parent–subsidiary organizations are corporate organizations consisting of a parent firm and its fully owned legally distinct subsidiary firms. The parent itself may be an operating concern or a holding company. Most conglomerates and multinational corporations are parent–subsidiary organizations. However, under our definition, a multinational or conglomerate that operates as a single legal entity articulated into divisions based on product line or geography would not be a parent–subsidiary organization. At the same time, a firm such as Xerox which is organized into legally distinct subsidiary firms operating in the same line of business (e.g., PARC in the case of Xerox) is a parent–subsidiary organization. Because subsidiaries are legally distinct, each can follow its own capital structure policy. Moreover, absent of explicit guarantees, non-tort related liabilities of any one firm in the parent–subsidiary organization are, in general, not liabilities of the other firms in the organization. Thus, insolvency of one firm in the organization does not imply the insolvency of the others. In fact, for subsidiary entities to issue debt that is not guaranteed by other firms in the parent–subsidiary organization is fairly frequent, as documented by Kolasinski (2009) and recognized by practitioners. Because the asset and liability structures of the constituent entities in a parent–subsidiary organization are different from each other the incentive to renegotiate claims, in general, will differ across the entities. Moreover, in the case of multinational firms, the legal systems in which claims would be renegotiated in the event of default also differ, possibly producing divergence in renegotiation incentives even when the component entities have similar asset–liability structures.

The aim of this paper is to develop a parsimonious theory of parent-subsidiary capital structure based on the flexibility generated by the legal boundaries between the constituent entities in a parent–subsidiary organization. The model will not explain why firms form subsidiaries or why firms finance investments using debt and equity. It will offer an explanation of how firms allocate debt obligations across subsidiaries and how parent–subsidiary organizations renegotiate debt claims if any of the debtor firms become distressed. The key insight of the model is that flexibility created by the parent-subsidiary structure generates renegotiation options that are costly ex ante but valuable ex post. This flexibility, which is alluded to in earlier empirical research, such as Slovin and Sushka (1997) and Vijh (2006) in the context of equity issuance, and Newberry and Dhaliwal (2001) and Desai et al. (2004) in the context of debt issuance, produces renegotiation options that lower ex post payments to creditors. Ex ante these options will be reflected in higher nominal interest rates attached to claims issued by the parent–subsidiary

1Lee Reicher, CPA, attorney and managing partner of the Los Angeles law firm Reish Luftman McDaniel & Reicher, says, “In addition to the tax benefits of forming a subsidiary, many companies crave the legal protections from potential plaintiffs and creditors it can provide. Liabilities and creditor claims of a subsidiary are ‘trapped’ in that subsidiary and can’t be passed on to the parent company. As a result, if the subsidiary runs into financial trouble, the parent company’s assets and its credit rating are protected.”

2La Porta et al. (1997) find significant differences in creditor protection across distinct commercial law traditions. La Porta et al. (1999) document the predictive power of corporate law for explaining world stock ownership patterns. Halet al. (2009) find a positive association between creditor protection and stock valuation as well as a negative association between creditor protections and stock price volatility. Fuentes and Maqueria (1999) provide evidence that, although Chilean commercial law is part of the French civil code tradition, which generally offers the weakest protection of creditor rights, the Chilean system, is, in fact, relatively creditor friendly. Desai et al. (2004) show that these legal-system differences explain important regularities in the capital structure of multinational firms. Antràs et al. (2009) develop a contract theory model of multinational investment not based on creditor rights.

3Slovin and Sushka (1997) find that firms act strategically and tend to issue equity in the parent/subsidiary whose equity is relatively overvalued; in contrast, Vijh (2006) finds no such strategic behavior on equity issuance.
organization; in the presence of agency conflicts, these higher interest rates will increase agency costs by distorting the incentives of inside owners. Because, claims issued to outsiders are correctly priced, increased agency costs will lower insiders’ payoffs. Thus, ex ante, insiders have an incentive to minimize the value of their renegotiation options. Such obligations are minimized by “biased balancing” of liabilities across the entities in the parent–subsidiary organization. Some debt is issued by all of the component entities, even entities with low liquidation value operating in jurisdictions with weak creditor protection. However, more debt is loaded on entities where creditor protection is stronger and assets have higher liquidation value. The bias in issuance is ex ante reflected in the ex post likelihood of renegotiation. In fact, when legal systems vary with respect to creditor-friendliness but not efficiency, the likelihood of renegotiation in a given system is proportional to the creditor-friendliness of that system. Efficiency differences provide an additional bias in renegotiation toward more efficient legal regimes. Thus, the endogenous choice of capital structure not only eliminates but even reverses the incentive of parent–subsidiary organizations to renegotiate with weak unsecured creditors under weak legal regimes.

Our work, to our knowledge, is the first to model the capital structure choices of parent–subsidiary organizations qua parent–subsidiary organizations (i.e., based on their defining characteristic, legal differentiation). However, a number of researchers have developed models that describe a subset of parent–subsidiary organizations, and perhaps include other corporate structures. For example, Huizinga et al. (2008) develop a tax-based model of multinational capital structure choice. Also, Chowdhry and Nanda (1994), Chowdhry and Coval (1998), and John et al. (1991) model the effects of tax regime differentials on the choice between debt and equity for multinationals. Our model differs from these in that our analysis is not restricted to parent–subsidiary organizations with cross-border operations. Much empirical work and some theoretical research, (e.g., Goel et al. (2004), Kahn and Winton (2002), Maksimovic and Phillips (2002), Ozbas (2005), Scharfstein (1998) and Stumpf et al. (2003)) have devoted attention to conglomerate firms. In practice, conglomerates no doubt overlap substantially with parent–subsidiary organizations. However, being a conglomerate is neither a necessary nor a sufficient condition for being a parent–subsidiary organization. The subject of our research, the effects of subsidiary structure, overlaps with Bebchuk (2009). However, our focus is different. Bebchuk considers the social optimality of the renegotiation options generated by the parent–subsidiary organization structure. We take the parent-subsidiary structure as given and analyze its effect on corporate debt policy.

Our analysis produces a number of testable implications for researchers in international finance and capital structure. These implications are presented below:

- The optimal mixture of subsidiary debt and parent debt depends on the nature of the parent–subsidiary organization’s business.

Specifically, we show that the greater the liquidation value of the assets of the business, the larger the relative contribution of the subsidiary’s level of borrowing to the parent–subsidiary organization’s overall borrowing. For any legal regime, higher liquidation value of the assets
tends to reduce the value of the renegotiation option and, hence, create more debt capacity at the subsidiary level. For example, a subsidiary operating a software business may be more dependent on parent debt relative to a subsidiary operating a steel plant.

- The optimal mixture of subsidiary debt and parent debt depends on the relative creditor-friendliness of the legal regimes.

In particular, we show that the greater the relative creditor-friendliness of one of the legal regimes, the larger the relative contribution of that financial market to the parent–subsidiary organization’s overall borrowing. This implication is consistent with the empirical findings in Davydenko and Franks (2008) and Desai et al. (2004).

- The optimal mixture of subsidiary debt and parent debt depends on the overall financial health of the company.

Claim enforceability is positively related both to creditor-friendliness and to the relative balance between the size of claims in the parent and subsidiary capital markets. Changes in the financial condition of the parent–subsidiary organization change the terms of trade between these two factors. An increase in the firm’s financial health makes ex post exploitation of the weaker legal regime more attractive. To counter this incentive and lower the agency cost of debt, the parent–subsidiary organization will reduce its borrowing from the subsidiary’s capital market.

Next, we extend the analysis to consider how the parent–subsidiary organization should allocate scarce internal assets between the parent creditors and the subsidiary creditors. These assets act to collateralize and, thus, guarantee the debt claims. Our analysis shows that

- There exists a positive minimum fraction of subsidiary debt that will be guaranteed by the parent. The minimum fraction is higher when the subsidiary operates under a weak legal system.

This implication is consistent with the empirical findings in Kolasinski (2009). For example, Kolasinski finds that among U.S. firms, a large part of the outstanding subsidiary debt is not guaranteed by their parent’s assets.5

As well as having implications for a firm’s capital structure mix, the model produces a number of new testable comparative statics relating to the firm value. One such prediction is

- Holding mean creditor-friendliness fixed, increased variance in the creditor-friendliness between the subsidiary market and parent market leads to lower overall default spreads.

As well as making ex ante predictions regarding optimal choice of capital structure, the model makes strong predictions regarding the venue that the parent–subsidiary organization will choose ex post to restructure its debt in the event of financial distress. One might conjecture that parent–subsidiary organizations would prefer to restructure under the legal regime with the least protection for creditor rights, but the opposite is true – the parent–subsidiary organization will, in fact, design its initial capital structure so as to eliminate ex post incentive to over-utilize the weaker legal regime.6 This characteristic of the optimal capital structure mix leads to another testable implication:

5 Also, see Stumpp et al. (2003) for discussions on parent non-guaranteed subsidiary debt.

6 This result highlights the importance of analyzing ex post restructuring of endogenously determined capital structures. In fact, a partial equilibrium analysis treating the capital structure of the firm as fixed would come to exactly the opposite conclusion.
Given equilibrium capital structures, the odds that restructuring will occur at the parent level or the subsidiary level are directly proportional to the creditor-friendliness of these markets.

Our model is predicated on the assumption that parents are shielded from subsidiary liabilities that they have not guaranteed, and that the parent corporations can successfully defend such shields. Subsidiary bankruptcy not accompanied by parent company bankruptcy may be less common but is clearly possible and does occur. For example, the 2005 bankruptcy filing by Entergy New Orleans, after Hurricane Katrina, was not accompanied by a bankruptcy filing by the parent company, Entergy Corp. In this case, headquarters justified subsidiary-only filing as a tool for permitting the parent company, Entergy Corp, to finance the bankrupt subsidiary through debtor-in-possession (DIP) financing.

Our plan for establishing the theory underlying these predictions, inter alia, is as follows. In Section 2 we present the basic environment. In Section 3 we analyze the bargaining problem associated with restructuring the debt of troubled firms. Also in this section we identify and value the renegotiation options embedded in the financing mix. In Section 4 we characterize the optimal design of a parent–subsidiary organization’s capital structure policies under the assumption that the level of cash flows in the event of financial distress is certain. In Section 5 we model the effects of differences in creditor-friendliness on optimal debt allocation. In Section 6, uncertainty regarding the extent of distress is introduced and the effect of shocks on capital structure is analyzed. In Section 7 we model the effects of dissipative cost on optimal capital structure and ex post debt renegotiation strategies. In Section 8 we discuss other extension possibilities. Section 9 provides concluding remarks. Some of the more cumbersome results and an extensive numerical exercise are delegated to the appendix.

2 Model Preliminaries

We assume that the risk-free rate of interest is zero; all agents are risk neutral; capital markets are competitive. In this context, consider an investment project. Financing the project requires one-time investment of $I$ dollars. The right to undertake the project is owned by a subsidiary, $S$. A parent firm $P$ owns all of the subsidiary’s equity. The capital structure and investment policies of both parent and subsidiary are both determined by the parent firm’s shareholders, who act to maximize their own welfare. Because the owners of the parent group act as if they are

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7Consider West Coast media company, which formed a subsidiary a few years ago when it decided to enter a new line of business. The venture did not go well, and some creditors were forced to sue for payment of their bills, which were ultimately settled for less than full value. However, the media company itself was shielded from the creditors’ reach by virtue of the subsidiary arrangement. “It was only because we kept it (the new operation) separate that we confined the contagion to the unit and didn’t wind up infecting the rest of the company,” says the owner, who requested anonymity. Another interesting case is Germany’s Arcandor, which owns 53% of Thomas Cook. Arcandor has filed for bankruptcy protection after the German government rejected a request for loan guarantees. Arcandor said its bankruptcy filing covered German retailer Karstadt and its mail-order businesses. However, it added that Thomas Cook would “remain unaffected.”

8Parents are sometimes exposed to liabilities of subsidiaries, such as when courts, under certain conditions, ignore the legal separation between parent and subsidiary, (e.g., when the subsidiary was created to perpetrate a tort). In doing so, the court is said to “pierce the corporate veil.” Factors that the courts usually consider in such cases are concealment or misrepresentation by the parent company, manipulation of assets or liabilities to concentrate the assets or liabilities, and failure to observe corporate formalities in terms of behavior and documentation (see, e.g., Bicker (2006) and Thomson (1991)). Because our model concerns contracted financial liabilities rather than tort liabilities, this sort of veil piercing is not relevant to our analysis.
a single agent, we will often refer to this group simply as the “shareholder.” All cash flows from the project are realized at date 2. All cash flows come from the subsidiary’s project. Since there are no internal funds or assets in place, all project financing, of necessity, is external. Financial claims in our model are debt claims on cash flows. Funds can be raised both by the subsidiary’s issuing debt and by the parent’s issuing debt. The focus of the model is on finding the optimal mix between these two sources of financing.

We denote the funds raised by selling bonds to creditor-group \( S \) and creditor-group \( P \) by \( B_S \) and \( B_P \) respectively. Thus, undertaking the project requires that \( B_S + B_P \geq I \). Similarly, we denote the face value of debt raised by the subsidiary to finance the new project by \( k_S \) and call it “subsidiary debt.” We denote the face value of debt raised by the parent to finance \( S \)’s new project by \( k_P \) and call it “parent debt.” For creditors to at least break even from providing financing, it must be the case that \( B_S \leq k_S \) and \( B_P \leq k_P \). Also, let \( K \) denote the aggregate face value of all bonds issued; that is, \( k_S + k_P = K \). We assume that creditors of the subsidiary are distinct from creditors of the parent. In the event of financial distress, both subsidiary creditors and parent creditors act as a group – each maximizing the total payoff from their respective claims.

If debt is not renegotiated, cash flows are distributed based on absolute priority. The parent’s cash flow comes from a dividend paid by the subsidiary to the parent, which is equal to the residual cash flow remaining after satisfying the subsidiary creditors. Thus, the parent’s cash flow is subordinated to subsidiary debt. The dividend from the subsidiary is then divided between parent debt and equity, again based on absolute priority.\(^1\)

The sequence of actions is as follows: First, at date -2, the shareholder picks a capital structure. This capital structure consists of zero-coupon debt issued to creditor-group \( S \), and/or zero-coupon debt issued to creditor-group \( P \), and written on the cash flows of the respective entities. Next, at time -1, the shareholder picks an unverifiable ex ante effort level \( e \). This ex ante effort level has a non-pecuniary disutility of \( \delta(e) \) to the shareholder. The effort level affects the likelihood, \( \nu \), that financial condition is nondistressed as opposed to distressed.

We assume that the probability of the nondistressed state, \( \nu \), is increasing in the ex ante effort. More specifically, we assume that \( \nu \) is a strictly increasing function of effort, \( e \). We normalize this function so that \( \nu(e) = e \). We also posit a continuum of ex ante effort levels, \( e \in [\underline{e}, 1] \). The cost of ex ante effort is, represented by the function \( e \rightarrow \delta(e) \), which satisfies standard regularity conditions; that is, \( \delta(\underline{e}) = \delta'(\underline{e}) = 0, \delta'' > 0, \delta''' > 0, \delta'(1) = \infty \).

After the ex ante effort level is selected at time -1, financial condition is revealed at time 0. Financial condition is observable by all parties, but not verifiable (and thus not contractible). This financial condition includes information on the state of the world and specific information on future cash flows at time 2. If the condition is distressed, continued operation as a going

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\(^9\)In our setting, this assumption that all equity is inside equity held by a controlling group of shareholders is sensible because, as will be apparent later, capital structure is designed to maximize the ex post effort incentives of inside owners. Such incentives will be attenuated by the existence of outside equity. In a richer model, one featuring private information on the part of some of the subsidiary-agents, for example, it will be easy to motivate outside equity participation at the subsidiary level.

\(^10\)In an earlier version, we permitted \( P \) to hold a senior claim on the assets of \( S \). We have dropped this feature from the current draft for the following reasons: (a) No new insights were provided by this feature of the analysis. In fact, we showed that, in equilibrium, senior claims in \( S \) and parent-company debts are perfect substitutes. (b) The inclusion of senior claim greatly complicates the analysis. (c) The legal doctrine of equitable subordination, common to a large number of legal systems, makes enforcement of the priority of debt claims held by an agent that also owns all the equity in a project, problematic.
concern will require further effort from the shareholder. If effort is not applied at time 1, assets will have to be liquidated for $\ell$ dollars at time 2. If this additional effort is applied, the total cash flow at time 2, the going-concern value, will equal $x \in [\underline{x}, \bar{x}]$.

The cost of ex post effort to the shareholder is $c$. If the state is nondistressed, then the cash flow will equal $x(\text{ND})$ and ex post effort is not required. Because no additional effort is required in the nondistressed state, lowering debt liabilities through renegotiation does not improve effort incentives. Thus, it is not possible to renegotiate claims in the absence of distress. Therefore, to simplify our analysis, we assume the firm is solvent in the nondistressed state, that is, $x(\text{ND}) \leq K$.

In contrast, we assume that, if distress occurs, the shareholder may attempt debt contract renegotiation. We assume that $\bar{x} - c - \ell > 0$, which implies that renegotiating the debt, rather than liquidating assets, is always the first-best policy. Finally, we assume that $\bar{x} - c < I$. Because the face value of debt must at least equal $I$, the assumption that $\bar{x} - c < I$ implies that claims must be renegotiated in the event of distress. This assumption is not required for our subsequent result, but the assumption simplifies that analysis by ensuring that renegotiation must take place in all distressed states.

Next, we incorporate “legal regime” into our analysis. Legal systems vary in the degree to which they favor creditors over debtors.\textsuperscript{11} We model the bias in the legal regime toward the creditor with a single index of creditor bargaining power denoted by $\rho$. This index, which is explained in detail below, represents the likelihood that the creditor will capture the surplus from renegotiation. Creditor-preferred debt, as defined in Harris and Raviv (1995), corresponds to $\rho = 1$, while debtor-preferred debt in their terminology corresponds to $\rho = 0$. Thus, this approach to modeling creditor rights is quite standard. In the initial part of the analysis, we will assume that the parent and subsidiary operate under the same legal regime; hence, the index of creditor bargaining power is same for both. Later, we examine the effect of jurisdictional variation by varying the creditors’ bargaining power. To summarize, we impose the following restrictions on the model’s parameters:

\begin{align*}
x(\text{ND}) &> \bar{x}, \\
\bar{x} &> \underline{x}, \\
\bar{x} &< c + K, \\
\underline{x} &> c + \ell, \\
x(\text{ND}) &\leq K.
\end{align*}

The timeline for the model is presented in figure below.

\textbf{2.1 Payoffs}

Recall that $k_P$ represents face value of the debt issued to creditor-group $P$, and $k_S$ represents face value of the debt issued to creditor-group $S$. Thus, in the distress state the contracted payments to creditor-group $S$, creditor-group $P$, and the shareholder, are given as follows: The

\textsuperscript{11}This observation is consistent both with the direct evidence provided by studies of automatic stay provisions in the United States and violations of creditor priority, such as Bhandari and Weiss (1996); international studies of credit spreads and legal systems, such as Desai et al. (2004); and standard models of the US bankruptcy system, such as Brown (1998).
payoff to creditor group $S$ in the distress state absent renegotiation is

$$\pi_S(z, k_S, k_P) = \min[z, k_S],$$

(6)

where $z$ represents cash flow either from liquidation ($\ell$) or operation ($x$). The payoff to creditor group $P$ is

$$\pi_P(z, k_S, k_P) = \min[(z - k_S)^+, k_P],$$

(7)

where $(z - k_S)^+ = \max[0, z - k_S]$. For example, in the event of liquidation $(z - k_S)^+ = \max[\ell - k_S, 0]$. The payoff to the shareholder, denoted by $\pi_M$, is the residual payment, after both creditor group $S$ and creditor group $P$ are satisfied; that is,

$$\pi_M(z, k_S, k_P) = z - \pi_S(z, k_S, k_P) - \pi_P(z, k_S, k_P).$$

(8)

If the firm is liquidated, that is, $z = \ell$, then $\pi_M = 0$. To simplify notation, we use $\bar{\pi}_D$ and $\bar{\pi}_P$ to represent the payoffs to creditor-group $S$ and creditor-group $P$ if the state is distress and the renegotiations fail:

$$\bar{\pi}_S = \min[k_S, \ell],$$

$$\bar{\pi}_P = \ell - \min[k_S, \ell] = \ell - \bar{\pi}_S.$$

Given our assumption (3), we know that $\bar{x} - K = \bar{x} - k_S - k_P < c$. Also, from equation (8) we know that $\pi_M = x - k_S - k_P$ cannot be greater than $c$, because $x \leq \bar{x}$; thus, if at least one of the debt claims is not restructured, the shareholder will not exert effort and liquidation will follow. In Section 3 we discuss in detail all possible debt renegotiation strategies available to the firm in the distress state.

### 3 Debt Renegotiation Strategies

In the distress state, the shareholder approaches each creditor group sequentially, with the shareholder choosing the sequence. After approaching a creditor group, the shareholder can either repudiate or affirm the outstanding debt contract of that group. If the shareholder affirms the debt payment with a given creditor group, the initial face value of the debt stands. If the shareholder repudiates a given debt contract, a bargaining game between the shareholder and the given creditor group ensues.

The game has a simple structure: First, the shareholder proposes a new claim structure to the creditor group. If the creditor-group accepts this offer, the claim structure is changed accordingly. If an initial offer by the shareholder is rejected, negotiations break down with
probability $1 - \rho$. If negotiations break down, all agents receive their payoffs in the event of default, $\pi_j$ where $j = S$ or $P$. If negotiations do not break down, the creditor-group being negotiated with makes a final offer. The rejection of this offer triggers liquidation.\(^\text{12}\) Note that if $\rho = 1$, then the creditor-group being negotiated with is able to make a final offer without risk of dissipation of value, and thus can capture all of the surplus from renegotiation. In contrast, if $\rho = 0$, rejection of the shareholder’s offer triggers dissipative liquidation. Thus, the shareholder can capture all of the surplus from avoiding liquidation. Once negotiations with the first creditor end, the shareholder may “never return” to renegotiate claims against that creditor.\(^\text{13}\)

Assuming the first negotiation did not result in liquidation, the shareholder turns to the next creditor; again, he can either repudiate or affirm the claim of this creditor. Negotiations with the second creditor follow the same schema as negotiations with the first creditor. Once negotiations cease, the shareholder makes his ex post effort decision and the situation proceeds as in the timeline specified above. The cash flow to the shareholder is positive only if both creditors’ (possibly renegotiated) claims are satisfied. Therefore, negotiating the debt level down to a point at which both creditors’ claims can be satisfied is a necessary condition for successful renegotiation. The shareholder’s payoff in a restructuring that ensures effort is the total cash flow less the (possibly renegotiated) claims, and it must be at least equal to the cost of effort, $c$.

Assuming the shareholder has affirmed the claim of one of the creditor groups, and that the face value of the affirmed claim is $k^r$, then the most the other creditor group can obtain in negotiations with the shareholder is $x - k^r - c$. The least that any creditor group can obtain is its liquidation value, $\pi_j$. For example, if $k_S > \ell$, then $\pi_P = 0$; that is, creditor-group $P$ receives no repayment if liquidation happens. Our assumptions ensure that the actual result of renegotiation is a weighted average between $x - k^r - c$ and $\pi_j$ with the weight on the upper end point fixed by $\rho$. In other words, the payment to a given creditor is $\rho (x - k^r - c) + (1 - \rho) \pi_j$, where $\rho \in (0, 1)$ is the index of creditor bargaining power.\(^\text{14}\)

Given our assumption that $x - c - \ell > 0$, the shareholder will always attempt to renegotiate at least one claim. This leaves the shareholder with three viable renegotiation strategies: Repudiate and renegotiate both claims; affirm creditor-group $S$’s claim and repudiate and renegotiate creditor-group $P$’s claim; affirm creditor-group $P$’s claim and repudiate and renegotiate creditor-group $S$’s claim. Ex post, at the time of renegotiation, lowering creditor claims weakly increases total future cash flows (increasing ex post effort incentives) and strictly increases the shareholder’s share of the cash flows. Thus, the shareholder will choose the strategy that minimizes payments to its creditors.

\(^{\text{12}}\)This bargaining game is thus a two-move version of the Osborne and Rubinstein (1990) bargaining game. Note that if the shareholder’s offer is rejected the shareholder will receive nothing because either (a) value is dissipated or (b) the shareholder is the second mover in an ultimatum game. Thus, the shareholder will never make an offer that will not be accepted. At the same time, there is no reason for the shareholder to make a higher offer than necessary for acceptance. Thus, the shareholder will always make the lowest offer the creditor will accept. Our formulation is the simplest bargaining game that produces a non-trivial division of the surplus.

\(^{\text{13}}\)The “never return” assumption is not required to obtain any of our results. As long as we follow the standard assumption of extensive bargaining games, that repeated rounds of negotiations are costly, we obtain exactly the same results. However, allowing for repeated returns to the same creditor in negotiations makes the analysis more cumbersome. For further analysis of this issue see Appendix A3.

\(^{\text{14}}\)A formal bargaining model, similar to the Osborne and Rubinstein (1990) model supporting this division of value, is developed in Appendix A1 of the paper for the case $\rho_S \neq \rho_P$, where $\rho_j$ represents the probability that a rejected offer by the $j$-th creditor-group does not trigger value dissipation.
3.1 Negotiating With Both Creditor Groups

We assume that the shareholder negotiates sequentially with both creditor groups. So, either the shareholder first negotiates with creditor-group $S$ and then negotiates with creditor-group $P$, or the shareholder first negotiates with creditor-group $P$ and then negotiates with creditor-group $S$. We show in Appendix A1 that the aggregate debt repayment is unaffected by the order of negotiation. Hence, we can assume that the shareholder first goes to creditor-group $S$ and then negotiates with creditor-group $P$.

Because the shareholder negotiates first with creditor-group $S$, creditor-group $S$ can obtain $x - c - \pi_P$ if they reject the shareholder’s initial offer and they are able to make a final counteroffer for the entire surplus. Remember, creditor-group $S$ is able to make this counteroffer with probability $\rho$. Otherwise, creditor-group $S$ forecasts that rejection will trigger liquidation, in which case creditor-group $S$ receives only the liquidation value, $\pi_S$. Thus, the expected value of rejection to creditor-group $S$ is given by

$$\rho (x - c - \pi_P) + (1 - \rho) \pi_S,$$

which is the expected value the shareholder must offer creditor-group $S$ in order for the offer to be accepted. If the shareholder’s offer is rejected, the shareholder will receive nothing. Thus, the best response of the shareholder is to offer creditor-group $S$ the expected value of rejection: Hence, creditor-group $S$ will receive an offer equal to the value of rejection and creditor-group $S$ will accept this offer, resulting renegotiated debt payment, $k^r_S$, of

$$k^r_S = \rho (x - c - \pi_P) + (1 - \rho) \pi_S$$

in any subgame perfect equilibrium. The first creditor’s claim must be satisfied in order for the shareholder to receive a positive payoff; thus, the first creditor’s claim is satisfied in any successful renegotiation. A final offer by the second creditor, then, must leave enough value both for the shareholder to receive a payoff of $c$ and for the first creditor to be satisfied. Thus, the creditor-group $P$ will receive

$$k^r_P = \rho (x - c - k^r_S) + (1 - \rho) \pi_P.$$

Simplifying this expression, we note that $\pi_S + \pi_P = \ell$ yields the aggregate repayment if the shareholder negotiates with both creditor groups. Formally,

$$C^*(x, \rho, c, \ell \mid \text{Both}) = (\rho(1-\rho) + \rho)(x - c) + (1 - \rho)^2 \ell = (1 - \eta(\rho))(x - c) + \eta(\rho) \ell,$$

where $\eta(\rho) = (1 - \rho)^2$. Note that, if $k_S > \ell$, then both parties make some concessions; hence, $k^r_S < k_S$ and $k^r_P < k_P$. If $k_S \leq \ell$, then renegotiating both claims is not a feasible strategy and, hence, only creditor-group $P$ makes a concession.

3.2 Negotiating With Only One Creditor Group

Next, we consider the cases where the shareholder affirms one debt claim and repudiates and renegotiates the other claim. Obviously, the shareholder has two possible options: Affirms
creditor-group S’s claim and repudiates and renegotiates creditor-group P’s claim or affirms creditor-group P’s claim and repudiates and renegotiates creditor-group S’s claim. Each of these cases is discussed in Sections 3.2.1 and 3.2.2, respectively.

### 3.2.1 Renegotiating Only With creditor-group P

If the shareholder decides to affirm creditor-group S’s claim and to repudiate only creditor-group P’s claim, then the payoff to creditor-group S is the issued face value, $k_S$. Of course, this option is feasible only if $x - c > k_S + \pi_P$; otherwise, creditor-group P will block the negotiation and take the project to liquidation. But the fact that $x - c - \ell > 0$ implies that $x - k_S - c$ can be negative only if $k_S > \ell$. If $k_S \leq \ell$, then $k_S + \pi_P = \ell$, implying that $x - c$ is always greater than $k_S + \pi_P$; that is, renegotiating with creditor-group P only is always a viable option.

Assuming it is viable, that is, $x - c > k_S$, the payoff to creditor-group P is the outcome of the following bargaining game: Creditor-group can obtain $x - c - k_S$ with probability $\rho$ if creditor-group P rejects the shareholder’s initial offer and makes a counter-offer for the entire surplus. Dissipation occurs with probability $1 - \rho$, in which case rejection triggers liquidation. Thus, for reasons identical to those given above, the shareholder offers creditor-group P the value of rejection and creditor-group P accepts the offer. Thus, the value of renegotiated debt is

$$k^*_P = \rho(x - k_S - c) + (1 - \rho)\pi_P = \rho(x - k_S - c) + (1 - \rho)(\ell - \pi_S).$$  \hspace{1cm} (13)

Note that if $k_S \geq \ell$, then $\pi_P = 0$; hence, $k^*_P$ reduces to $\rho(x - k_S - c)$. If $k_S \leq \ell$, then $(x - k_S - c)$ is always no less than $\pi_P$, implying that $k^*_P \geq \pi_P$. Hence, the aggregate debt repayment if the shareholder affirms creditor-group S’s claim and only repudiates creditor-group P’s claim, is

$$C^*(x, \rho, c, \ell | renegotiate only with creditor-group P) = (1 - \rho)k_S + \rho(x - c) + (1 - \rho)(\ell - \pi_S).$$  \hspace{1cm} (14)

### 3.2.2 Negotiating only with creditor-group S

Similarly, the shareholder’s strategy to affirm creditor-group P’s debt and to repudiate and renegotiate with creditor-group S is feasible if $x - k_P - c - \pi_S > 0$; otherwise, creditor-group S will block the negotiation and take the project to liquidation. The intuition is straightforward: In any renegotiation process creditor-group S is always assured of its reservation value, $\min[\ell, k_S]$. If $k_S \leq \ell$, then $x - c - k_P - k_S < 0$, given assumption (3). Thus, if $k_S \leq \ell$, then negotiating only with creditor-group S is not a plausible strategy. The payoff to creditor-group S is the payment on the renegotiated debt claim given by

$$k^*_S = \rho(x - k_P - c) + (1 - \rho)\pi_S = \rho(x - k_P - c) + (1 - \rho)\ell.$$  \hspace{1cm} (15)

Hence, the aggregate debt repayment if the shareholder affirms creditor-group P’s claim and only repudiates creditor-group S claim,

$$C^*(x, \rho, c, \ell | negotiate only with creditor-group S and k_S > \ell) = (1 - \rho)k_P + \rho(x - c) + (1 - \rho)\ell. \hspace{1cm} (16)$$
3.3 Minimum Aggregate Debt Repayment

The shareholder maximizes his distress state payoff by minimizing distress state aggregate debt repayment. Thus, the aggregate payoff to both creditor-groups is given by the following expression:

\[ C^*(x, \rho, c, \ell, k_S, k_P) = \min \left[ \begin{array}{l}
(1 - \eta(\rho))(x - c) + \eta(\rho)\ell, \\
(1 - \rho)k_S + \rho(x - c) + (1 - \rho)(\ell - \bar{\pi}_S), \\
(1 - \rho)k_P + \rho(x - c) + (1 - \rho)\bar{\pi}_S.
\end{array} \right] \] \tag{17}

The three terms in the minimum expression on the right-hand side of equation (17) represent the distress state aggregate debt repayments under different renegotiating strategies adopted by the shareholder. Note that in the absence of any deadweight cost, the renegotiation strategy that minimizes ex post debt repayment also maximizes the ex post shareholder’s payoff in the distress state. We show in Section 7 that if the renegotiation process involves incurring some deadweight cost, then there is a divergence between “repayment minimizing” and “shareholder’s payoff maximizing” debt renegotiation strategies.

3.4 Optimal Ex post Exploitation of Renegotiation Options

Expression (17) describes the payoffs generated by the renegotiation options presented by the shareholder’s choice of capital structure. Ex post renegotiation of contracts presents the shareholder with the following basic tradeoff: Repudiating a debt contract forces negotiations between the shareholders and the creditor-group whose claim is repudiated. Regardless of whether a claim is repudiated and renegotiated, or is not repudiated and remains at the original terms, the claims of one creditor group have an indirect effect on the payoff of the other creditor group. Each dollar claimed by one creditor-group lowers the funds available to the other creditor-group by an equal amount. Similarly, each dollar extracted from one group strengthens the position of the other; for example, if the bargaining power of the other creditor is equal to \( \rho \), this implies that each dollar of concession extracted by the shareholder in negotiating with one of the creditor-group only produces \( 1 - \rho \) in gains to the shareholder.

As the going-concern value, \( x \), and the creditor bargaining power, \( \rho \), increase, only one claim can profitably be renegotiated by the shareholder. The choice of claim to renegotiate depends on the relative size of these claims. The payoff from renegotiating only creditor-group \( P \)'s claim is given by the second term of expression (17). Similarly, the payoff from renegotiating only creditor-group \( S \)'s claim is given by the third term of expression (17). Ceteris paribus, repudiating the larger claim generates the most debt relief. These observations have important consequences for predicting the shareholder’s optimal ex post renegotiation strategy; hence, these observations are formally developed in Lemma 1 below.\(^{15}\)

Lemma 1.

1. The likelihood of renegotiating a creditor-group’s claim is increasing in the initial face value of the creditor group’s claim (\( k \)).

\(^{15}\)To make the conclusions of this lemma both precise and easy to state, we require the following definition. For a given model parameter and creditor restructuring policy, we shall say that an increase (decrease) in model parameter makes it more (less) likely that policy will be followed if the set of other parameters over which the given policy is optimal increases (decreases) with increases in the parameter.
2. The larger the going-concern value after restructuring, \( x \), the more likely that restructuring will occur only at one level (subsidiary or parent). The smaller the going-concern value after restructuring, the more likely that restructuring will occur both at the parent and the subsidiary levels.

3. If \( k_P > k_S + \ell \), then the shareholder will always renegotiate the claims of creditor-group \( P \). If \( k_S > k_P - \ell \), then the shareholder will always renegotiate the claims of the creditor-group \( S \).

4. If \( k_P = k_S + \ell \) the shareholder is indifferent between the strategies of negotiating only with creditor-group \( P \) and only with creditor-group \( S \).

**Proof.** Follows directly from inspecting expression (17).

The conclusions of Lemma 1 are illustrated in Figure 1. Note that as the going-concern value increases across the distress range, the shareholder moves from a policy of complete debt restructuring at both parent and subsidiary levels to a policy of restructuring only at the subsidiary level or only at the parent level. If \( k_P > k_S + \ell \), then the shareholder will either negotiate with both groups or will negotiate only with creditor-group \( P \). If \( k_S > k_P - \ell \), then the shareholder will either negotiate with both groups or will negotiate only with creditor-group \( S \). Thus, it is always optimal for the shareholder to repudiate and renegotiate the “largest” outstanding claim, where the size of the claims is adjusted for the superior collateral position of the subsidiary firm.

![Figure 1](image-url)

Figure 1: This figure depicts aggregate debt repayment in the distress state as a function of the going-concern value, \( x \). We fix \( \rho = 0.60 \), \( c = 0.20 \) and \( \ell = 0.10 \). The going-concern value in the event of financial distress, \( x \), is varied between 1.00 and 2.45. In panel \( A \) we fix \( k_S = 1.45 \) and \( k_P = 1.00 \), and in panel \( B \) we fix \( k_S = 1.00 \) and \( k_P = 1.45 \). The solid lines, in both panel \( A \) and \( B \), represent the aggregate debt repayment if the shareholder repudiates and renegotiates both debt claims. The dashed lines, in both panels \( A \) and \( B \), represent the aggregate debt repayment if the shareholder affirms creditor-group \( P \)’s claim and renegotiates creditor-group \( S \)’s claim. The dashed-dotted lines, in both panel \( A \) and \( B \), represent the aggregate debt repayment if the shareholder affirms creditor-group \( S \)’s claim and renegotiates creditor-group \( P \)’s claim. The shareholder will choose the renegotiation strategy that minimizes aggregate ex post debt repayments in the distress state for different going-concern values, \( x \), as shown in the figure by the gray highlight.

## 4 Repayment Maximizing Capital Structures

From the previous subsection, we see that borrowing at both the parent level (creditor-group \( P \)) and the subsidiary level (creditor-group \( S \)) creates renegotiation options for the shareholder (or
the debtor). This section characterizes the impact of these renegotiation options on the optimal capital structure of the firm.

To understand the firm’s optimization problem, first note that the capital markets are competitive. This implies that the creditor-groups on average will break even when purchasing the firm’s securities. Thus, any agency costs will be borne by the shareholder (or we can say the firm). For this reason, the firm will design its initial capital structure to minimize agency costs. As we have seen in the previous section, renegotiation of the debt claim at date 0 ensures that the ex post agency problem will not induce any agency costs. In contrast, the deviations of the ex ante date -1 effort choice from the first-best effort choice will create agency costs.

What is the source of these agency costs? Recall that the sole effect of ex ante effort choice is to reduce the likelihood of financial distress. If the firm bore the entire reduction in total cash flow that occurs in financial distress, the shareholder would select first-best effort ex ante. However, ex post debt forgiveness, due to renegotiation, transfers some of the costs of suboptimal effort to the creditors. Because of this possibility of transfer, the shareholder’s ex ante choice of effort level will be less than the first-best effort level. This attenuation of effort induces agency costs. These agency costs are proportional to the amount of debt forgiveness the firm can negotiate ex post. For this reason, the firm, in order to mitigate agency costs, attempts, at date -2, to design its capital structure to minimize the debt forgiveness ex post. In other words, we assert that optimal financial designs always maximize, for any fixed total nominal level of debt issued, the total expected payments to creditors in the event of financial distress. This assertion is hardly surprising given that the basic logic behind this claim can be traced to Jensen and Meckling (1976) and Myers (1977). However, the proof of the assertion is a bit tedious and forces us to develop notation not used anywhere else in the paper. Thus, we defer the proof of this result to Appendix A2.

Next, we shall characterize the capital structures that maximize the expected payments by the firm in the event of financial distress or, equivalently, minimize the expected shortfall faced by the firm in the event of financial distress. Henceforth, we shall simply call the expected shortfall in the event of financial distress the “expected shortfall.” We shall show that, in general, a mix of parent-level borrowing (i.e., borrowing from creditor-group \( P \)) and subsidiary-level borrowing (i.e., borrowing from creditor-group \( S \)) is optimal for the firm. Because all optimal financing policies minimize the expected shortfall, all optimal financial policies must solve the problem \( SP(K | \rho) \), for some \( K \). \( SP(K | \rho) \) is defined below:

\[
\max_{k_P \geq 0, k_S \geq 0} C(k_S, k_P); \quad k_S + k_P = K,
\]

where \( C(k_S, k_P) = \int C^*(x, k_S, k_P) G(dx) \) is the expected value of creditor claims in the event of financial distress and \( C^* \) is defined by expression (17). The solution to problem \( SP(K | \rho) \) will be the focus of our subsequent analysis.

4.1 Characterizing Optimal Policies

We initiate our analysis by considering a simplified version of \( SP(K | \rho) \) in which the cash flow, conditional on the realization of the distressed state, is deterministic. We call this case the con-
ditional certainty case and represent the corresponding maximization problem with \( SP_F(K \mid \rho) \).

We shall show that optimal financing policies exist and that optimal policies always feature a mix between subsidiary and parent-level financing. Additionally, the cost of capital is fairly “flat” in that the optimal mix between parent level debt and subsidiary level debt is frequently not unique. However, it is always the case that the set of optimal subsidiary–parent debt mixtures is a proper closed interval in the interior of the range of feasible mixes. In some cases, this interior interval reduces to a single point. To develop these results, let \( x^o \in [x, \bar{x}] \) represents the cash flow when distress occurs. Using this definition, we can express the shortfall minimization problem, in the special case where going-concern cash flows in distress are deterministic, as follows:

\[
\max C^*(x^o, k_S, k_P)
\]

\( k_P \geq 0, \ k_S \geq 0 \)

\( k_S + k_P = K. \) \hspace{1cm} (19)

Note that the creditors’ payoffs are concave in financing policies. Moreover, the feasible set of payments is convex. Thus, the set of optimizers of \( SP_F(K \mid \rho) \) is also convex. Solving for these optimal policies allows us to explicitly calculate the aggregate creditors’ payoffs in the distressed state, \( C^* \). These insights are formally stated in Lemma 2.

**Lemma 2.** The maximum of \( SP_F(K \mid \rho) \), and thus the maximum total payoff to creditors in financial distress under the optimal financing policy, is given by

\[
\max(SP_F(K \mid \rho)) = \max C^* = \max \left[ (1 - \rho)^2 \ell + \rho \left( 2 - \rho \right) (x^o - c), \ (1 - \rho) \left( \frac{K + \ell}{2} \right) + \rho (x^o - c) \right].
\]

**Proof.** See the proof stated in Section A4.1.

To understand the implications of Lemma 2, first note that the first term in the minimum expression defining creditor payoffs represents the payoff to creditors when the firm renegotiates both claims; the second term in the minimum operator used to define \( C^* \) represents the payoff to the creditors when the firm renegotiates only one of the contracts. Because optimal policies call for minimizing the switching option, this payoff is the same regardless of which creditors’ contracts are renegotiated. Both branches of the minimization function represent weighted averages. Since both \( 1 - \rho \) and \( (1 - \rho)^2 \) are decreasing in creditors’ bargaining power, this monotonicity implies, not surprisingly, that total creditor payoff is always increasing in creditors’ bargaining power. These insights are formally stated in Corollary 1 below.

**Corollary 1.** For any given initial value of creditors’ bargaining power,

1. Increasing creditors’ bargaining power, \( \rho \), increases aggregate debt repayment in the distress state, \( C^* \).

2. Increasing creditors’ bargaining power, \( \rho \), decreases the range of values of \( x \) over which the shareholder negotiates with both creditor groups.

3. Also, increasing the liquidation value, \( \ell \), the face value of the aggregate debt, \( K \), and going-concern value, \( x^o \), increases the aggregate debt repayment in the distress state, \( C^* \).
4. Increasing the costs of restructuring, c, decreases aggregate debt repayment in the distress state.

Proof. It follows from inspection of the creditor-payoff function \( C^* \) given in Lemma 2.

Figure 2 depicts the effect of creditor-friendliness on aggregate debt repayment. Not surprisingly, as the creditor-friendliness of a legal regime increases, that is, as \( \rho \) increases, so does the aggregate debt repayment in the distress state. Perhaps what is not obvious is that as the creditor-friendliness increases, the range of \( x \) over which the firm renegotiates with both groups diminishes. This is because, as \( \rho \) increases, \((1 - \rho)^2\) decreases at a much faster rate than \( 1 - \rho \). Thus, the firm switches from negotiating with both groups to negotiating with one group, at lower \( x \), in order to minimize the ex post aggregate debt repayment.

Figure 2: This figure depicts minimum aggregate debt repayment in the distress state as a function of going-concern value, \( x \). We fix \( k_P = 1.275, k_S = 1.175, c = 0.20, \ell = 0.10 \). We consider two legal regimes: A creditor-friendly regime (\( \rho = 0.65 \)) and a relatively less creditor-friendly regime (\( \rho = 0.55 \)). As the creditor-friendliness of a legal regime increases, so does the aggregate debt repayment, \( C^* \), in the distress state. The solid segment of each line represents the going-concern values such that the shareholder finds it optimal to repudiate and renegotiate only one debt claim – either creditor-group \( S \)'s claim or creditor-group \( P \)'s claim. The dashed line of each line represents the going-concern values such that the shareholder finds it optimal to repudiate and renegotiate both debt claims. As the creditor-friendliness increases from 0.55 to 0.65, the relative size of the solid segment increases; this implies that the shareholder is more likely to repudiate and renegotiate only one debt claim under a relatively more creditor-friendly legal regime.

Although at \( \rho = 1 \), the degenerate case, the firm is again indifferent between negotiating with any one group or both groups, the aggregate debt repayment is always equal to \( x - c \) if \( \rho = 1 \), irrespective of the initial financing choice. Similarly, if \( \rho = 0 \), another extreme legal regime, the firm always negotiates with both groups and the aggregate debt repayment is always equal to \( \ell \), irrespective of the initial financing choice.

Next, consider the optimal financial policies. Optimal financial policies are the ones that maximize total creditor payoff in the event of financial distress. Obviously, these policies are the solution to problem \( SP_F(K \mid \rho) \). We formally state the optimal financial policies (solutions to problem \( SP_F(K \mid \rho) \)) in Theorem 1 below.

**Theorem 1.** The set of solutions to \( SP_F(K \mid \rho) \), \( \text{Argmax}(SP_F(K \mid \rho)) \), is characterized as follows: If (i) creditors’ bargaining power is below the cutoff level, \( \rho^* \), the maximum of aggregate debt repayment is flat, featuring a range of optimal debt allocations between parent and
subsidiary; if (ii) creditors’ bargaining power exceeds this cutoff level, the optimal debt mix is unique. That is,

\[ \text{i. If } \rho < \rho^* = \frac{1}{2} (K - \ell) \]
\[ \Rightarrow \text{Argmax}(SP_F(K | \rho)) = \{(k_S^+, k_P) : (k_S, k_P) = \lambda (k_S^+, k_P^+) + (1 - \lambda) (k_S^-, k_P^-)\}, \]
\[ (k_S^-, k_P^-) = \{\ell + \rho (x^o - c - \ell), K - \rho (x^o - c - \ell) - \ell\}, \]
\[ (k_S^+, k_P^+) = \{K - \rho (x^o - c - \ell), \rho (x^o - c - \ell)\}, \]
\[ \lambda \in [0, 1]. \]

\[ \text{ii. If } \rho \geq \rho^* = \frac{1}{2} (K - \ell) \]
\[ \Rightarrow \text{Argmax}(SP_F(K | \rho)) = \{(k_S^*, k_P^*)\}, \]
\[ k_S^* = K + \frac{\ell}{2}, \]
\[ k_P^* = K - k_S^*. \]

**Proof.** See the proof stated in Section A4.2. \( \square \)

Theorem 1 shows that the set of optimal capital structure is usually large. When the creditors’ bargaining power is low \((\rho < \rho^*)\), optimal financing policies lead to the renegotiation of both claims. When both claims are being renegotiated, the only restriction on financing policies is that the firm must not allocate so little value to creditor-group \(S\) that ex post it is not optimal to renegotiate with creditor-group \(S\). A large range of policies produce this result; the set of optimal policies consists of a line segment with endpoints, \(\{(k_S^-, K - k_S^+), (k_S^+, K - k_S^+)\}\). When the creditors’ bargaining power is low the firm enjoys financing flexibility in the sense that it can choose its optimal capital structure from a large possibility set, \(k_S^* \in \{k_S^-, k_S^+\}\) and \(k_P = K - k_S^*\); whereas, when the creditors’ bargaining power is high \((\rho \geq \rho^*)\), optimal financing policies lead to the renegotiation of only one claim and the optimal capital structure consists of a single point, \((k_S^*, K - k_S^*)\).

The firm loses its financing flexibility as the optimal capital structure choice reduces to a single point. Although for both high and low creditors’ bargaining power, optimal financing policy minimizes the value of the firm’s ex post switching option. If the optimal mix is unique, the optimal mix equates the payoff from subsidiary-only and parent-only restructuring options, making the ex post choice of sequencing a worthless option for the firm.

Next, we turn to the comparative statics of the optimal solution. Because optimal initial allocations are a set of values rather than a single value, we must define what it means for a set to increase or decrease. We choose one simple definition: An interval increases if both of its end points weakly increase with increase in the parameter value and vice versa.\(^{16}\) Theorem 2 uses this definition to characterize optimal financing policies.

\(^{16}\)This definition is a simple special case of the induced set ordering defined in Milgrom and Shannon (1994) and Topkis (1998).
Figure 3: In this figure panels A and B depict the firm’s optimal capital structure when its total nominal face value debt $K = 1$. The horizontal axis in each panel represents the nominal debt payment promised to creditor-group $S$, $k_S$; hence, the promised payment to creditor-group $P$ is $1 - k_S$. The vertical axis in each axis represents the actual total payout to creditors in the event of financial distress given that the going-concern value $x^o = 1$. In panel A, the bargaining power of both creditor-group $S$ and creditor-group $P$ is 0.4. In panel B, the bargaining power of both creditor-group $S$ and creditor-group $P$ is increased to 0.8. We fix $c = 0.10$ and $\ell = 0$. In both panels, the solid lines represent the aggregate payoff to creditors in the event only creditor-group $S$’s claim is restructured; the dashed lines represent the aggregate payoff from restructuring only creditor-group $P$’s claim; and the dotted-dashed lines represent the aggregate creditors’ payoff when both claims are restructured. Because the shareholder decides on the optimal pattern of restructuring, the actual payout will be the minimum of these three restructuring options. This minimum is represented in both panels by a thick gray highlight. Note that, as shown in Theorem 1, when creditors’ bargaining power is relatively low, a range of debt mixes is optimal, giving the firm financing flexibility. This range of optimal capital mixes is given by the interval $\{(k_S^-, K - k_S^-), (k_S^+, K - k_S^+)\}$ in panel A. At the higher levels of creditor bargaining power depicted in panel B, a unique optimal policy, $(k_S^*, K - k_S^*)$, exists.

**Theorem 2.** The optimal financing policies are characterized by the following:

1. Value maximizing ratios between subsidiary and total debt are interior. That is, if $k_S \in \text{Argmax}(SP_F(K \mid \rho))$, then $\frac{k_S}{k_S + k_P} \in (0, 1)$.

2. As creditors’ bargaining power, $\rho$, increases, financing flexibility decreases; that is, the set $\{k_S^-, k_S^+\}$ shrinks.

3. As the going-concern value, $x^o$ increases, financing flexibility decreases.

4. The optimal level of debt issued to each creditor exceeds the liquidation value of the creditor’s claim (i.e., $k_S > \ell$).

**Proof.** See the proof stated in Section A4.3.

5 Effect of Disparate Legal Systems on Optimal Debt Allocation

We consider the case where the parent company operates under a legal regime that is more creditor-friendly than the legal regime under which the subsidiary operates; that is, we assume that creditor-group $S$ has lower bargaining power vis-à-vis creditor-group $P$.\(^\text{17}\) We denote the index of creditors’ bargaining power of creditor-group $P$ by $\rho_P$ and the index of creditors’ bargaining power of creditor-group $S$ by $\rho_S$. Then our assumption implies that $\rho_P > \rho_S$. Given

\(^\text{17}\) Again, the implications for capital structure choice are qualitatively similar when you consider that the parent operates under a legal regime that is more debtor-friendly vis-à-vis the subsidiary.
our assumptions and the bargaining game described in Appendix A1, the payoff to creditors is given by the following expression:

$$C^*(x, c, \ell, \rho_S, \rho_P, k_S, k_P) = \min \left[ \eta(\rho_S, \rho_P) \ell + (1 - \eta(\rho_S, \rho_P))(x - c), \right.$$

$$\left. k_S + \rho_P (x - k_P - c) + (1 - \rho_P) \bar{\pi}_P, \right.$$ 

$$\left. k_P + \rho_S (x - k_S - c) + (1 - \rho_S) \ell, \right)$$

where

$$\eta(\rho_S, \rho_P) = (1 - \rho_S)(1 - \rho_P).$$

Repudiating a debt contract forces negotiations between the shareholder and the creditor group whose claim is repudiated. The outcome of these negotiations depends on the structure of the legal regime under which each creditor group operates. The legal regime determines the maximum rents the creditor can ensure for herself in the bankruptcy process.

What renegotiation path should the firm pursue when $\rho_S$ is not equal to $\rho_P$? First, suppose both $k_S$ and $k_P$ are large and the creditor-friendliness of their respective legal systems is weak; that is, both $\rho_S$ and $\rho_P$ are relatively small. Then, the first term in the minimum expression given in (20) is the smallest, and both creditors will be forced to make concessions. Similarly, both creditors are likely to make concessions when going-concern cash flows, $x$, are small relative to the size of creditors’ original claims, $K$. As going-concern value and creditor bargaining power increase, only one claim can profitably be renegotiated by the firm. The choice of claim to renegotiate depends on the nature of the legal regime, and the relative values of $\rho_S$ and $\rho_P$ and $k_S$ and $k_P$. As shown in Section 3.4, repudiating the larger claim generates the most debt relief. However, if the larger claim is in the more creditor-friendly legal regime, then such creditors may be able to extract so much more in post-repudiation negotiations that it is better to repudiate the smaller claim. These observations have important consequences for predicting the firm’s optimal ex post renegotiation strategy. These consequences are formally developed in Lemma 3 below.
Lemma 3. When the subsidiary and the parent operates under disparate legal systems, the likelihood of renegotiating a creditor group’s claim is increasing in the initial face value of the creditor group’s claim ($k$) and decreasing in the creditor-friendliness of the legal regime ($\rho$) under which the creditor group operates. The larger the going-concern value after restructuring, $x$, the more likely that restructuring will occur only at the subsidiary level. The smaller the going-concern value after restructuring, $x$, the more likely that restructuring will occur at both the parent and the subsidiary levels.

Proof. Follows directly from inspecting equations (20) and (21).

The conclusions of Lemma 3 are illustrated in Figure 5 below. Note that as the going-concern value increases across the distress range, the firm moves from a policy of complete debt restructuring at both parent and subsidiary levels to a policy of restructuring at only the subsidiary level. The parent level restructuring is optimal in the intermediate range.

![Figure 5: This figure depicts creditor payoffs under the three renegotiation systems. The fixed parameters used to generate the figure are as follows: $k_S = 1.00$, $k_P = 1.45$, $\rho_S = 0.50$, $\rho_P = 0.75$, $c = 0.20$, $\ell = 0.10$. The going-concern value in the event of financial distress, $x$, is varied between 2.00 and 2.45. The solid line represents the total payoff to creditors from the policy of renegotiating with both creditor-group $S$’s claim and creditor-group $P$’s claim. The dashed line represents the creditor payoff from renegotiating only with creditor-group $S$’s claim. The dashed-dotted line represents the creditor payoff from renegotiating only with creditor-group $P$’s claim. The firm will choose the policy that minimizes total payments to creditor; the minimum debt repayment for different going concern value, $x$, is shown by the dark gray line.](image)

5.1 Disparate Legal Systems: Repayment Maximizing Capital Structures

Next, we characterize the capital structures that maximize the expected payments by the firm in the event of financial distress, given $\rho_S < \rho_P$. As in Section 4.1, we argue that because all optimal financing policies minimize the financial shortfall, all optimal financial policies must solve the problem $SD(\bar{x}, \ell, K, \rho_S, \rho_P)$, for some $K$. $SD(\bar{x}, \ell, K, \rho_S, \rho_P)$ is defined below:

$$\max_{k_P \geq 0, k_S \geq 0} C(k_S, k_P | \bar{x}, \rho_S, \rho_P);$$

$$k_S + k_P = K,$$

(22)
where \( C(k_S, k_P|\rho_S, \rho_P) = \int C^*(x, k_S, k_P|\rho_S, \rho_P) G(dx) \) and \( C^* \) is defined by expression (20). Again, we initiate our analysis by considering a simplified version of \( SD(\tilde{x}, \ell, K, \rho_S, \rho_P) \) in which the cash flow, conditional on the realization of the distressed cash flow state, is deterministic. As in Subsection 4.1, we call this the conditional certainty case in a disparate legal environment, and represent the corresponding maximization problem with \( SD_F(x^o, \ell, K, \rho_S, \rho_P) \), where \( x^o \) represents the cash flow when distress occurs. Optimal financing policies call for more utilization of capital markets operating under legal systems that provide stronger protection of creditors’ rights. Additionally, although an optimal capital structure exists, the cost of capital is fairly “flat” in the optimal mix between parent level debt and subsidiary level debt. Again, note that the creditors’ payoffs are concave in financing policies and the feasible set of payments is convex. Thus, the set of optimizers of \( SD_F(x^o, \ell, K, \rho_S, \rho_P) \) is also convex. Solving for these optimal policies allows us to explicitly calculate the aggregate creditors’ payoffs in the distressed state, \( C^* \). These insights are formally stated in Lemma 4.

**Lemma 4.** The maximum of \( SD_F(x^o, \ell, K, \rho_S, \rho_P) \), and thus the maximum total payoff to creditors in financial distress under the optimal financing policy, is given by

\[
\max(SD_F(x^o, \ell, K, \rho_S, \rho_P)) = \min \left( \eta^*(\rho_S, \rho_P) \ell + (1 - \eta^*(\rho_S, \rho_P))(x^o - c), \phi^*(\rho_S, \rho_P) \left( \frac{1}{2}(\ell + K) \right) + (1 - \phi^*(\rho_S, \rho_P))(x^o - c) \right),
\]

where \( \eta^* \) and \( \phi^* \) are weighting functions varying between 0 and 1. These functions are dependent on the creditor-friendliness of the two legal systems and are explicitly defined as follows:

\[
\eta^*(\rho_S, \rho_P) = (1 - \rho_S)(1 - \rho_P), \quad \phi^*(\rho_S, \rho_P) = \frac{(1 - \rho_S)(1 - \rho_P)}{\frac{1}{2}((1 - \rho_S) + (1 - \rho_P))}.
\]

**Proof.** See the proof stated in Section A4.4. \( \Box \)

Recall that because optimal policies call for minimizing the switching option, this payoff is the same regardless of which of the creditors’ contracts is renegotiated. Both branches of the minimization function represent weighted averages; the weight of the smaller term in the average is given by either \( \phi^* \) or \( \eta^* \). Since both \( \phi^* \) and \( \eta^* \) are decreasing in creditor bargaining power, this monotonicity implies, not surprisingly, that total creditor payoff is always increasing in creditor bargaining power. Perhaps less obvious is that, for a fixed total level of creditor bargaining power, total creditor payoffs are maximized when the bargaining power is unevenly allocated between the two countries. These insights are formally stated in Corollary 1 below.

**Corollary 2.** For any given initial distribution of creditor bargaining power,

1. Increasing the bargaining power, \( \rho \), of either creditor group, while holding the bargaining power of the other creditor group fixed, increases total creditor payoffs in the distress state, \( C^* \).

2. Moreover, uniformity in creditor groups’ bargaining power leads to lower total creditor payoffs than extreme allocations of bargaining power: Consider creditor payoffs under two
Proof. It follows from inspection of the creditor-payoff function \( C^* \) given in Lemma 4.

Next, consider the optimal financial policies. Optimal financial policies are the ones that maximize total creditor payoffs in the event of distress. Obviously, these policies are the solution to problem \( SP_F(K, \rho) \). In formalizing the theorem, we use \( \bar{\rho} = \frac{1}{2}(\rho_S + \rho_P) \). Thus, \( \bar{\rho} \) is the average bargaining power of the creditors operating under two disparate legal systems. We formally state the optimal financial policies (solutions to problem \( SP_F(K, \rho_S, \rho_P) \)) in Theorem 3 below.

**Theorem 3.** The set of solutions to \( SP_F(K, \rho_S, \rho_P) \), \( \text{Argmax}(SP_F(K, \rho_S, \rho_P)) \), is characterized as follows:

i. If \( \frac{\bar{\rho}}{1-\bar{\rho}} < \rho^*_p = \frac{\frac{1}{2}(K-\ell)}{x^o - c - \ell} \)

\[ \Rightarrow \text{Argmax}(SP_F(K, \rho_S, \rho_P)) = \{(k_S, k_P) : (k_S, k_P) = \lambda(k^+_S, k^+_P) + (1-\lambda)(k^-_S, k^-_P)\}, \]

where

\[ (k^+_S, k^+_P) = (\ell + \rho_S (x^o - c - \ell), K - \rho_S (x^o - c - \ell) - \ell), \]

\[ (k^-_S, k^-_P) = (K - \rho_P (x^o - c - \ell), \rho_P(x^o - c - \ell) ). \]

\[ \lambda \in [0, 1] \]

ii. If \( \frac{\bar{\rho}}{1-\bar{\rho}} \geq \rho^*_p = \frac{\frac{1}{2}(K-\ell)}{x^o - c - \ell} \)

\[ \Rightarrow \text{arg max}(SP_F(K, \rho_S, \rho_P)) = \{(k^*_S, k^*_P)\}, \]

where

\[ k^*_S = \frac{(1-\rho_S)K + (\rho_S - \rho_P)(x^o - c) + (1-\rho_S)\ell}{(1-\rho_S) + (1-\rho_P)} , \]

\[ k^*_P = K - k^*_S. \]

**Proof.** See the proof stated in Section A4.5.

Like Theorem 1, Theorem 3 confirms that the set of optimal polices is usually fairly large. When the average creditors’ bargaining power is low \( \bar{\rho} < \rho_p^* \), condition (i) of Theorem 3 holds. Note that \( \bar{\rho} < \rho_p^* \) even though one creditor group, say group \( j \), has bargaining power \( \rho_j \) greater than \( \rho_p^* \). In this case, optimal financing policies lead to the renegotiation of both claims. When both claims are being renegotiated, the set of optimal policies consists of a line segment with endpoints \( (k^-, k^+) \). When the average creditors’ bargaining power is high \( \bar{\rho} \geq \rho_p^* \), condition (ii) of Theorem 3 holds. In this case, optimal policies lead to the renegotiation of only one claim and the optimal capital structure policy consists of a single point. Contrary to Theorem 1, where legal systems are uniform, the optimal capital structure under disparate legal systems is a function of going-concern value, \( x \). Figure 6 depicts the implications of Theorem 3 for different ranges of parameter values. It also contrasts the outcome vis-à-vis the uniform creditors’ bargaining power case.
Figure 6: In this figure, panels A and B depict the optimal capital structure when the firm’s total nominal face value debt $K = 1.00$. The horizontal axis in each represents the nominal debt payment promised to creditor-group $S$, $k^*_S$; hence, the promised payment to creditor-group $P$ is $1.00 - k^*_S$. The vertical axis in each represents the actual total payout to creditors in the event of financial distress given that the going-concern value $x^o = 1.00$. In panel A, the bargaining power of creditor-group $S$ and creditor-group $P$, namely $\rho_S$ and $\rho_P$, are 0.40 and 0.50 respectively. In panel B, the bargaining power of creditor-group $S$ and creditor-group $P$ is increased from 0.40 to 0.60 and from 0.50 to 0.70 respectively. We set $c = 0.10$ and $\ell = 0.00$. As in Figure 3, the solid lines represent the payoff to creditors in the event only parent-level debt is restructured; the dotted lines represent the payoff to creditors from restructuring both parent and subsidiary-level debt; the dashed lines represent the total creditor payoff when only subsidiary debt is restructured. Because the manager decides on the optimal pattern of restructuring, the actual payout to each creditor group will be the minimum over these three restructuring options. This minimum is represented in both panels by a thick gray line. Note that, as shown in Theorem 1, when creditor bargaining power is relatively low, a range of debt mixes is optimal. This range of optimal polices is given by the interval $[k^*_S, k^+_S]$ in panel A. At the higher levels of bargaining power depicted in panel B, a unique optimal policy, represented by $k^*_S$, exists.

Next, we turn to the comparative statics of the optimal solution. We show that the firm’s utilization of a capital market is proportional to the creditor-friendliness of that market, but a more creditor-friendly market will not be the sole financing venue. Because optimal initial allocations are a set of values rather than a single value, we must define what it means for a set to increase. We choose one simple definition: An interval increases if both of its end points weakly increase with increase in the parameter value.\textsuperscript{18} Theorem 4 uses this definition to characterize optimal financing policies.

**Theorem 4.** The optimal financing policies are characterized by the following:

1. Even for disparate legal systems ($\rho_S < \rho_P$), value maximizing ratios between subsidiary and total debt are interior. That is, if $k \in \text{Argmax}(SP_F(K|\rho))$, then $k_S/(k_S + k_P) \in (0, 1)$.

2. Creditor-group $S$’s claim is increasing (decreasing) in creditor-group $S$’s bargaining power. Similarly, creditor-group $P$’s claim is increasing (decreasing) in creditor-group $P$’s bargaining power. More than half of total financing is always obtained at the parent-level.

**Proof.** See the proof stated in Section A4.6. \hfill \Box

The intuition behind Theorem 4 is straightforward: When designing its capital structure, the firm is attempting to maximize the repayment rate on its nominal obligations. Given that a creditor’s marginal ability to capture value is positively related to that creditor’s bargaining

\textsuperscript{18}This definition is a simple special case of the induced set ordering defined in Milgrom and Shannon (1994) and Topkis (1998).
power and inversely related to the size of the creditor’s claim, increasing a creditor’s bargaining power raises the marginal value of that creditor’s claim in extracting payments from the firm and thus increases the degree to which the firm will rely on that creditor when formulating the initial financing mix.

6 Effect of Uncertain Distress State Cash Flow

As in uniform legal regime case, the key to solving $SP_U(K, \rho)$ is to compute the going concern values that lead to the exercise of each of the renegotiation options. Performing the algebra to find the points at which the values of two of three negotiation options are equal yields:

$$x_{bh}(k_S, k_P) = c + \frac{1}{\rho_S} k_S - \frac{1 - \rho_S}{\rho_S} \ell,$$

$$\hat{x}_{bs}(k_S, k_P) = c + \frac{1}{\rho_P} (k_P + \ell) - \frac{1 - \rho_P}{\rho_P} \ell;$$

$$\hat{x}_{sp}(k_S, k_P) = c + \frac{(1 - \rho_S) + (1 - \rho_P)}{\rho_P - \rho_S} k_S$$

$$+ \frac{1 - \rho_S}{\rho_P - \rho_S} (k_P + k_S + \ell).$$

(23)

$\hat{x}_{sp}(k_S, k_P)$ represents the going-concern value at which renegotiating only with creditor-group $S$ produces the same total debt payoffs as renegotiating only with creditor-group $P$. Again, our assumptions imply that the slope of the creditor’s payoff in $x$ is highest if the firm negotiates with both creditors, and most flat if the firm negotiates with only creditor-group $S$. This slope relationship implies that if only creditor-group $P$’s debt restructuring is ever optimal, it must be optimal for intermediate values of $x$. Optimality for intermediate values is possible only if the payoff in $x$ from creditor-group $P$’s debt restructuring meets the payoff from a total restructuring at a lower value of $x$ than the value of $x$ at which creditor-group $S$’s debt restructuring meets the payoff from both companies’ restructuring. This observation permits a simple description of the regions over which the different renegotiation sequences are optimal:

$$\hat{x}(k_S, k_P) = \max\{\min[x_{bp}(k_S, k_P), x_{bs}(k_S, k_P)], x\}$$

$$\hat{x}(k_S, k_P) = \min[\max[x_{bs}(k_S, k_P), x_{sp}(k_S, k_P)], \hat{x}].$$

(24)

Note that if $x < \hat{x}$, then both creditor-groups’ debt contracts will be renegotiated; if $x \in (\hat{x}, \hat{\hat{x}})$, then only debt of creditor-group $P$ will be restructured; if $x > \hat{\hat{x}}$, then only debt to creditor-group $S$ will be restructured. Using these facts, we can write the expected creditor payoff in the event of financial distress as follows:

$$C(k_S, k_P) = \int_{\chi}^{\hat{x}(k_S, k_P)} \left( (\eta(\rho_S, \rho_P) \ell + (1 - \eta(\rho_S, \rho_P))(x - c) \right) G(dx)$$

$$+ \int_{\hat{x}(k_S, k_P)}^{\hat{\hat{x}}(k_S, k_P)} \left( k_S (1 - \rho_P) + \rho_P (x - c) \right) G(dx)$$

$$+ \int_{\hat{\hat{x}}(k_S, k_P)}^{\hat{x}} \left( k_P (1 - \rho_P) + \rho_S (x - c) + (1 - \rho_S) \ell \right) G(dx).$$

(25)
Using this definition and basic optimization theory, we can provide the following characterization of the optimal financial policy design under uncertain cash flows. This is formally stated in Theorem 5 below.

**Theorem 5.** Optimal financial policies exist; these policies always feature a positive level of both subsidiary and parent-level debt. Moreover, there always exists an optimal policy featuring a higher proportion of debt financing at the parent-level. The optimal policy, given \( k_P + k_S = K \), is explicitly defined by the conditions

\[
(1 - \rho_P) \left( G(\hat{x}(k_S, K - k_S)) - G(\hat{x}(k_S, K - k_S)) \right) - (1 - \rho_S) \left( 1 - G(\hat{x}(k_S, K - k_S)) \right) = 0.
\]

**Proof.** See the proof stated in Section A4.7.

The tradeoff behind the first-order condition in Theorem 5 is that an increased reliance on creditor-group S’s debt, \( k_S \), increases the total debt payment for all cash flow levels in the interval \((\hat{x}, \hat{x})\) over which the firm renegotiates only with creditor-group P’s creditor. At the same time, increasing creditor-group S’s debt lowers the nominal payments to creditor-group P and lowers actual debt repayment in the high cash flow states (i.e., states where \( x > \hat{x} \)) over which the firm negotiates only with creditor-group S’s creditor. The marginal costs of these two effects are equated by the adjustments in the two intervals, \((\hat{x}, \hat{x})\) and \((\hat{x}, \hat{x})\), induced by changing the capital structure mix. As the level of subsidiary debt increases, the interval \((\hat{x}, \hat{x})\) shrinks and \((\hat{x}, \hat{x})\) grows. Similarly, increasing subsidiary debt increases the interval \((\hat{x}, \hat{x})\) and shrinks \((\hat{x}, \hat{x})\). Thus, at some intermediate point, the marginal benefits associated with the two intervals are equalized.

The interesting comparative static issue raised by introducing differences in the creditors’ bargaining power and cash flow uncertainty regards the magnitude of financial distress in the allocation of restructuring across capital markets. In the deterministic cash flow model, the firm sets its capital structure in a fashion that eliminates the option value of choosing between creditor-group S’s and creditor-group P’s capital markets. Thus, in this setting, it is not possible to make any deterministic prediction as to which market will be the more likely venue for restructuring. Once uncertainty is introduced, it is no longer possible to equate value, so that for any given realization of going concern value, the firm will usually have a strict preference for one of the two venues. The term \( G(\hat{x}) - G(\hat{x}) \) represents the probability that the firm will opt for restructuring creditor-group P’s debt, and the term \( 1 - G(\hat{x}) \) represents the likelihood that the firm will opt for restructuring creditor-group S’s debt. Rearranging the first-order condition given in Theorem 6 yields the following immediate corollary.

**Corollary 3.** Given an optimal mix of subsidiary and parent-level debt, the odds ratio between an ex post restructuring at parent-level against subsidiary-level (given by \( (G(\hat{x}) - G(\hat{x}))/ (1 - G(\hat{x})) \)) equals \((1 - \rho_S)/(1 - \rho_P)\), implying that

1. Parent-level restructuring is always at least as likely as subsidiary-level restructuring;
2. The odds of a parent-level (subsidiary-level) restructuring are increasing (decreasing) in the bargaining power of creditor-group P and decreasing (increasing) in the bargaining power of creditor-group S.
7 Optimal Capital Structure with Dissipative Cost

In our base model, both legal systems are efficient in the sense that there is no "deadweight" cost associated with involving the legal systems with our firm’s problem. In this section, we extend our analysis by assuming that there is a cost which we call the "dissipative cost," that the creditors/firm must pay out of the corpus if they involve their legal system with the debt restructuring process. For simplicity, we normalize the dissipative cost to zero for the legal system under which creditor-group $P$ operates. We assume a positive dissipative cost, $d$, for the legal system under which creditor-group $S$ operates. In particular, we assume that creditor-group $S$’s rejection of the firm’s initial reorganization proposal triggers a cost associated with delay, and that the cost is paid out of the corpus of the firm. Our analysis shows that the existence of dissipative cost produces strong predictions regarding the negotiation strategies of the shareholder.

7.1 Debt Renegotiation with Dissipative Cost

First, we consider what happens when creditor-group $S$’s claim is renegotiated after the shareholder affirms creditor-group $P$’s claim. If creditor-group $S$ rejects the initial reorganization offer of the shareholder and makes a counter-offer, such counter offer obviously will set the shareholder on its reservation value. But because of the delay in the renegotiation process, the corpus has already been impaired and the value of the restructured firm is reduced to $x - d$ from $x$. Further, this delay in the process of renegotiation impairs the liquidation value too, and the liquidation value is reduced to $\ell - d$ from $\ell$. We assume that $\ell - d > 0$.

Creditor-group $S$’s minimum payoff in the event of liquidation is $\ell - d$. And because the bankruptcy code states that each party in a reorganization must receive at least as much as it would in a liquidation (otherwise, it is deemed to have rejected the plan), the smallest offer creditor-group $S$ will accept from the shareholder is equal to its expected payoff from rejecting the shareholder’s offer, which is

$$\rho_S(x - c - d - k_P) + (1 - \rho_S)(\ell - d).$$

Hence, the aggregate debt repayment, if the shareholder affirms creditor-group $P$’s debt and repudiates and renegotiates creditor-group $S$’s debt, is

$$\rho_S(x - c - d) + (1 - \rho_S)k_P + (1 - \rho_S)(\ell - d) = \rho_S(x - c) + (1 - \rho_S)(k_P + \ell) - d.$$ (27)

Thus, the payoff to the shareholder from renegotiating only with creditor-group $S$ is

$$\text{Equilibrium restructured firm value} - \text{original parent debt} - \text{renegotiated subsidiary debt} = x - c - \rho_S(x - c) - (1 - \rho_S)(k_P + \ell) + d = (1 - \rho_S)(x - c) - (1 - \rho_S)(k_P + \ell) + d.$$ (28)

Note that we do not subtract the dissipative cost, $d$, from the equilibrium value of the restructured firm, because the dissipative cost is not paid in equilibrium. We will derive conditions such that the shareholder will never find it optimal to renegotiate creditor-group $S$’s claim.
Next, we consider what happens when creditor-group $P$’s claim is renegotiated after the shareholder affirms creditor-group $S$’s claim. Given our assumption that there is no dissipative cost involved in the process of renegotiating with creditor-group $P$’s claim, the aggregate debt repayment is

$$\rho_P(x - c - k_S) + k_S = \rho_P(x - c) + (1 - \rho_P)k_S. \quad (29)$$

Thus, the shareholder’s payoff from renegotiating only with creditor-group $P$ is

$$\text{Equilibrium restructured firm value} - \text{original subsidiary debt} - \text{renegotiated parent debt} = x - c - \rho_P(x - c) - (1 - \rho_P)k_S = (1 - \rho_P)(x - c) - (1 - \rho_P)k_S. \quad (30)$$

When the firm repudiates both debt claims, the delay in the process of renegotiation with creditor-group $S$ impairs the corpus as well as the liquidation value. The maximum payment to the first creditor in the event agreement is reached decreases by the dissipative cost, $d$, and the maximum that can be offered is $x - c - d$. Thus, the first creditor’s payoff, which is the weighted average of these maximum and minimum values, increases by committed assets in place. The same holds for the second creditor in the negotiation sequence. Similarly, in the counterfactual situation in which no claims are renegotiated, dissipative cost has no impact on creditors’ payoff. Hence, the aggregate debt repayment if the shareholder negotiates with both creditor-groups is

$$(1 - (1 - \rho_S)(1 - \rho_P))(x - c) - (1 - \rho_S)\ell - d. \quad (31)$$

Hence, the shareholder’s payoff from renegotiating with both creditor-groups is

$$\text{Equilibrium restructured firm value} - \text{renegotiated subsidiary debt} - \text{renegotiated parent debt} = (1 - \rho_S)(1 - \rho_P)(x - c) - (1 - \rho_S)\ell. \quad (32)$$

Note that when the shareholder renegotiates both creditor-group $P$’s debt and creditor-group $S$’s debt, the equilibrium value of the restructured firm is lower, $x - c - d$ instead of $x - c$. Thus, the aggregate payoff to the shareholder is given by the following expression:

$$\max \left[ \begin{array}{c}
(1 - \rho_S)(1 - \rho_P)(x - c) - (1 - \rho_S)\ell, \\
(1 - \rho_P)(x - c) - (1 - \rho_P)k_S, \\
(1 - \rho_S)(x - c) - (1 - \rho_S)(k_P + \ell) + d.
\end{array} \right] \quad (33)$$

The three terms in the maximum expression on the right-hand side of expression (33) represent the shareholder’s payoff under three different debt renegotiation strategies. Note that in the presence of dissipative cost, the debt renegotiation strategy that maximizes the shareholder’s payoff does not necessarily minimize aggregate debt repayment in the distress state. This is because some strategies may allow the firm to avoid incurring the dissipative cost and may increase both shareholder’s payoff and debt holders’ payoff in such a way that the aggregate increase in payoffs equals $d$. But one thing we know for certain: The debt negotiation strategy that the shareholder adopts, ex post, is the one that maximizes the shareholder’s payoff.
7.2 Optimal Capital Structure

Given that the risk-free rate is zero and all capital markets are competitive, both creditor groups on average need to break even. Thus,

\[ e(K, C, d)K + (1 - e(K, C, d))C = I, \]  

where \( I \) is the required level of investment, \( C \) is the aggregate debt repayment in the distress state, and \( K \) is the promised debt repayment in the nondistress state. The value of the function \( e(K, C, d) \), the normalized effort, is the likelihood of being in the nondistress state.

We also know that ex post, if we are in the distress state, the shareholder chooses the debt renegotiation strategy that maximizes its payoff. The shareholder’s payoff from repudiating and renegotiating both debt claims is independent of the shareholder’s choice of capital structure; but the shareholder’s payoff from repudiating and renegotiating only one debt claim is affected by the shareholder’s choice of initial capital structure. When the shareholder negotiates only one debt claim, the aggregate debt repayment in the distress state, \( C \), is either \( \rho_P(x - c) + (1 - \rho_P)k_S \) or \( \rho_S(x - c) + (1 - \rho_S)(k_P + \ell) - d \). Suppose \( (1 - \rho_P)(x - c) - (1 - \rho_P)k_S > (1 - \rho_S)(x - c) - (1 - \rho_S)(k_P + \ell) + d \), which implies that in the distress state the shareholder will repudiate creditor-group \( P \)’s claim and affirm creditor-group \( S \)’s claim, which further implies that debt repayment in the distress state is \( \rho_P(x - c) + (1 - \rho_P)k_S \). Now suppose the shareholder transfers \( \Delta k \) from creditor-group \( P \) to creditor-group \( S \) in such a way that the sign of the inequality is preserved; that is,

\[ (1 - \rho_P)(x - c) - (1 - \rho_P)(k_S + \Delta k) > (1 - \rho_S)(x - c) - (1 - \rho_S)(k_P - \Delta k + \ell) + d. \]  

(35)

This transfer decreases the shareholder’s payoff in the distress state; but the transfer increases the distress state aggregate debt repayment from \( \rho_P(x - c) + (1 - \rho_P)k_S \) to \( \rho_P(x - c) + (1 - \rho_P)(k_S + \Delta k) \). Hence, for any given level of effort choice and dissipative cost, using equation (34), it reduces the debt repayment in the nondistress state, \( K \), which is equivalent to reducing the costs of borrowing. One can keep increasing \( \Delta k \) in a way such that the shareholder finds no need to alter its debt renegotiation strategy, but aggregate debt repayment in the distress state increases. Similarly, we can show that if \( (1 - \rho_P)(x - c) - (1 - \rho_P)k_S < (1 - \rho_S)(x - c) - (1 - \rho_S)(k_P + \ell) + d \), the shareholder can transfer capital from creditor-group \( S \) to creditor-group \( P \) and reduce \( K \).

Hence, the shareholder chooses \( k_S \) and \( k_P = K - k_S \) such that shareholder’s payoff from renegotiating only with creditor-group \( S \) is the same as its payoff from renegotiating only with creditor-group \( P \). Thus, the shareholder chooses \( k_S \) such that equations (28) and (30) are equalized; that is,

\[ (1 - \rho_P)(x - c) - (1 - \rho_P)k_S = (1 - \rho_S)(x - c) - (1 - \rho_S)(k_P + \ell) + d. \]  

(36)
This gives

\[ k^d_S = K - \frac{(K - \ell)(1 - \rho_P) + (x - c)(\rho_P - \rho_S) + d}{2 - \rho_P - \rho_S} \quad (37) \]

\[ k^d_P = K - k^d_S, \quad (38) \]

where \( k^d_S \) is the amount of capital raised initially from creditor-group \( S \) in the presence of dissipative cost. Similarly, \( k^d_P \) is the amount of capital raised from creditor-group \( P \) such that \( k^d_S + k^d_P = K \). What is the implication of this choice of \( k^d_S \) and \( k^d_P \) on effort choice? Using the results from Appendix 2 and equation (34) we know that the shareholder’s optimal effort choice is increasing in aggregate debt repayment in the distress state, \( C \). Because, from equation (36), we know that the shareholder’s payoff is the same whether the shareholder repudiates creditor-group \( S \)’s claim and affirms creditor-group \( P \)’s claim, or the shareholder repudiates creditor-group \( P \)’s claim and affirms creditor-group \( S \)’s claim, it must be the case that optimal debt repayment, when the shareholder renegotiates only one debt claim, is

\[ C^d = \max \left[ \rho_P(x - c) + (1 - \rho_P)k^d_S, \rho_S(x - c) + (1 - \rho_S)(k^d_P + \ell) - d \right] \quad (39) \]

when the shareholder finds it optimal to renegotiate with only one creditor group.\(^{19}\) The shareholder’s actual choice depends on the sign of the expression

\[ \rho_P(x - c) + (1 - \rho_P)k^d_S - \rho_S(x - c) - (1 - \rho_S)(k^d_P + \ell) + d; \quad (40) \]

substituting the values of \( k^d_S \) and \( k^d_P \) from equations (36) and (37) and simplifying expression (40), we obtain the following expression:

\[ \frac{d(1 - \rho_P) + (\rho_P - \rho_S)(1 - \rho_S)(K + \ell) + c - x}{2 - \rho_P - \rho_S}. \quad (41) \]

Expression (41) is not always positive. We need additional conditions: For example, if \( \rho_P = \rho_S \), then expression (41) is positive. But if expression (41) is positive, then there exists an equilibrium such that the shareholder never finds the optimal strategy to repudiate only creditor-group \( S \)’s claim. Even though the shareholder is indifferent (because the shareholder’s ex post payoff is the same), the aggregate debt repayment is higher if the shareholder adopts the strategy of repudiating creditor-group \( P \)’s claim and affirming creditor-group \( S \)’s claim as opposed to repudiating creditor-group \( S \)’s claim and affirming creditor-group \( P \)’s claim. If aggregate debt repayment in the distress state is higher, then by equation (34) we know that \( K \) is lower, and, hence, the equilibrium shareholder’s effort is higher. This result is formally stated in Theorem 6 below.

**Theorem 6.** The shareholder will never find it optimal to repudiate only creditor-group \( S \)’s claim if the dissipative cost \( d \geq d^* \), where

\[ d^* = \frac{(\rho_P - \rho_S)(1 - \rho_S)(x - c - K + \ell)}{1 - \rho_P}. \]

\(^{19}\)However, the optimal debt repayment is \( C^* = \min \left[ C^d, (1 - (1 - \rho_S)(1 - \rho_P)) (x - c) - (1 - \rho_S)\ell - d \right] \).
Proof. See the proof stated in Section A4.8.

The variable $d^*$ is the minimum dissipative cost required to keep the shareholder from adopting the strategy of renegotiating only with creditor-group $S$. The firm gives up the advantages of negotiating with creditor-group $S$ – the creditor group with lower bargaining power – in order to avoid paying the dissipative cost. If $d^*$ is negative, it is trivially smaller than $d$, which is, by our assumption, positive. Given our assumption that $x > c + \ell$, $d^*$ is always less than $\ell$. In Figure 7 we plot the relationship between $d^*$ and $x$ for given values of all other parameters.

![Figure 7: This figure depicts the relation between the minimum dissipative cost, $d^*$, and the distress state cash flow, $x$. We fix $\rho_P = 0.6$, $\rho_S = 0.40$, and $c = 0.2$. We fix the dissipative cost, $d = 0.10 < 0.20 = \ell$. The solid line depicts $d^*$ for $K = 1.00$. The dashed line depicts $d^*$ for $K = 1.25$. Even when $K = 1.00$ and we allow the distress state cash flow, $x$, to vary between 0.9 and 1.1, the firm never finds it optimal to renegotiate only with creditor-group $S$, because $d^*$ is lower than the actual dissipative cost for all values of $x$. When $K = 1.25$, $d^*$ is even more lower than $d$ for all values of $x$. For any $x$, the likelihood that the manager renegotiates only with creditor-group $S$ decreases with $K$.]

Next, we derive the impact of increasing the dissipative cost, $d$, on the optimal capital structure, $k^d_S$ and $k^d_P$. We differentiate equation (37) with respect to $d$ and we obtain

$$\frac{\partial k^d_S}{\partial d} = \frac{-1}{2 - \rho_P - \rho_S} < 0.$$ (42)

Since $k^d_P = K - k^d_S$, the sign of $\frac{\partial k^d_P}{\partial d}$ is $> 0$. Thus, as dissipative cost, $d$, increases the firm raises fewer funds from creditor-group $S$ and more funds from creditor-group $P$.

8 Extensions

Our analysis demonstrates that differences in legal systems can be exploited to develop a parsimonious model of the firm’s choice between credit markets. This model yields strong testable implications even in a setting lacking many of the frictions considered by earlier researchers. Legal systems theory can be further developed to provide convincing explanations of many other facets of financial management. Space constraints prevent a complete development of all of these issues here. However, in the subsequent subsections we will sketch two extensions of our base model and thereby extract further implications for legal systems and capital structures.
First, we will consider the extension whereby the parent company has assets in place, and subsequently we will consider the extension whereby the subsidiary credit market has some structural deficiencies that result in a deadweight cost associated with negotiations.

8.1 Parent’s Assets in Place and the Securitizing Subsidiary Debt

In our base model, the firm has no assets other than the investment project. This lack of assets in place allows us to abstract from issues relating to the allocation of internal capital and loan guarantees. Because the parent corporation has no assets in the base model, allocation policies are moot and parent guarantees are pointless. In this subsection, we shall extend our analysis by assuming that the firm has parent-country assets that it can commit to the creditors of the subsidiary. This analysis shows that the legal-system arbitrage perspective produces strong predictions regarding the allocation of internal capital by managers.

To extend the analysis, assume that the firm controls assets in place that yield a payoff of \( y \) dollars at date 2. This payoff is independent of the firm’s ex post and ex ante effort decisions. Like the payoff on the project in the subsidiary country, the payoff on assets in place is known at date 0, the date at which negotiations take place, but unknown at date -2, the date at which financing is obtained. From the perspective of date -2, the going concern value \( \tilde{x} \) and assets in place \( \tilde{y} \) are random variables. We assume that these random variables are non-negatively dependent; that is, for any increasing functions \( f \) and \( g \), \( \text{COV}[f(\tilde{x}), f(\tilde{y})] \geq 0. \)

At date -2, simultaneous with raising financing, a fraction, \( \alpha \), of assets in place is committed to the subsidiary and \( 1 - \alpha \) of assets in place is retained by the parent corporation. Assets committed to the subsidiary can be paid to subsidiary creditors in the event of contract renegotiations and will be captured by subsidiary creditors in the event of liquidation. We impose a debt covenant preventing the firm from agreeing to transfer parent-country assets in place to the subsidiary creditors, and vice versa. The only effect of assets in place on our analysis is to increase both the amount received by creditors in the event of negotiation breakdown and the amount creditors can demand from the firm when the firm decides to negotiate with them.

Thus, when the firm repudiates both debt claims, the lower bound on the first creditor-group’s offer, liquidation value, increases by committed assets in place. The maximum payment to the first creditor in the event agreement is reached also increases by committed assets in place. Thus, the first creditor’s payoff, which is the weighted average of these maximum and minimum values, increases by committed assets in place. The same holds for the second creditor in the negotiation sequence. Thus, when the firm renegotiates both contracts, an increase in assets in place by 1 dollar increases creditor payoffs by 1 dollar. Similarly, in the counterfactual situation in which no claims are renegotiated, an increase in assets in place has no impact on creditor payoffs.

Next consider what happens when one creditor-group’s claim is renegotiated after the other.

---

\( ^{20} \)In the statistics literature, what we term non-negative dependence is called positive dependence. Non-negative dependence includes independence as a special case. It implies, but is somewhat stronger than, the assumption of non-negative correlation, because non-negative correlation is consistent with a negative correlation between the random variables over a subset of values of the random variables. These sorts of negative correlations over subsets are ruled out by the assumption of non-negative dependence. See, for example, Shakad (1982) for more discussion of these issues.

\( ^{21} \)Note also that a complete development of a model with assets in place would also require that we modify the basic parameter assumptions (1) to (4) in the base model to account for the new factors introduced into the analysis. In the sketches provided in this section, these details will be omitted.
creditor-group’s claim is affirmed. Suppose, without loss of generality, that the parent-country creditors’ claim is renegotiated. Then the claim of the subsidiary creditors, because it is not renegotiated and is paid in full, is not affected by the allocation of assets in place. Now consider the parent-country creditors. The highest final debt offer that the firm will be willing to accept from the parent-country creditor is \( x + y - c - k_S \). At this payment level the firm is indifferent between providing the effort input. Thus, the final creditor offer will equal \( x + y - c - k_S \).\(^{22}\)

In the event of liquidation, the parent-country country creditor will receive committed assets in place of \( (1 - \alpha) y \). This implies the weighted average of the maximum and minimum payments equals \( (1 - \rho_P)(1 - \alpha)y + \rho_P(x + y - c - k_S) \). Using the same logic to derive payments to creditors for the other negotiation sequence and summing up the payments to the two creditors shows that the total payment to creditors will be given as follows in the new scenario:

\[
C^*(x, y, \alpha, k_S, k_P) = \min \left[ y + \eta(\rho_S, \rho_P) \ell + (1 - \eta(\rho_S, \rho_P))(x - c), \right.
\]

\[
 k_S (1 - \rho_P) + \rho_P (x - c) + (1 - \rho_P)(1 - \alpha) y, \]

\[
 k_P (1 - \rho_S) + \rho_S (x - c) + (1 - \rho_S) (\ell + \alpha y) \right].
\]

Let \( P \) represent the probability measure over the square induced by \( (\tilde{x}, \tilde{y}) \). Define

\[
C(k_S, \alpha) = \int_{\mathbb{R}^2} C^*(x, y, \alpha, k_S, K - k_S) dP(x, y).
\]

For exactly the same reasons as advanced in the previous sections, optimal policies satisfy the condition that no other policy produces a smaller shortfall for the same level of nominal debt payments. In other words, optimal policies \((k^*_S, k^*_P, \alpha^*)\) are solutions to the problem

\[
\max_{k_S \geq 0, k_P \geq 0, \alpha} C(k_S, \alpha).
\]

Let \( C_P(k_S, \alpha) \) represent the set of \((x, y)\) values over which parent level restructuring is selected ex post by the shareholder, and let \( C_S(k_S, \alpha) \) represent the set of values over which a subsidiary level restructuring will be selected ex post by the shareholder. Differentiating the objective function of \( (45) \) with respect to the control variables yields

\[
\frac{\partial C}{\partial k_S} = (1 - \rho_P) P[C_P(k_S, \alpha)] - (1 - \rho_S) P[C_S(k_S, \alpha)];
\]

\[
\frac{\partial C}{\partial \alpha} = -(1 - \rho_P) \int_{C_P(k_S, \alpha)} y dP(x, y) + (1 - \rho_S) \int_{C_S(k_S, \alpha)} y dP(x, y).
\]

By the same arguments as given in the previous sections, the mix between subsidiary debt and parent debt must be internal. Thus, equation \( (46) \) must vanish at the optimal policy. A rearrangement of the equations in the first-order conditions yields the following characterization

\(^{22}\)This payment is consistent with the covenant restricting debt payments by the following argument: Given the constraint that the firm cannot transfer assets in place already committed to the subsidiary creditor to the parent-country creditor, the most the firm is able to pay the parent-country creditor without violating the covenant is \( x + (1 - \alpha)y \). For the same reasons as advanced in the base model, the firm will always choose a capital structure in which the face value of each claim exceeds its liquidation value (Theorem 2 (d)). Thus, \( k_S > \alpha y \). The fact that \( k_S > \alpha y \) implies that the amount the firm is willing to concede in a final offer, \( x + y - c - k_S \), is less than the maximum amount permitted by the covenant, \( x + (1 - \alpha)y \).
of the optimal mix between subsidiary and home country debt:

\[
\frac{1 - \rho_P}{1 - \rho_S} = \frac{P[C_S(k^*_S, \alpha^*)]}{P[C_P(k^*_S, \alpha^*)]}. \tag{47}
\]

Equation (47) implies that the probabilities of subsidiary and parent debt restructuring are both positive at the optimal financial policy. Using equation (47) and the partial derivatives of the objective function, and the first-order condition for an interior optima, we see that the following characterization of the optimal fraction of value to allocate to the subsidiary holds:

\[
\begin{align*}
\alpha^* &= 0 \Rightarrow E_P[\tilde{y} \mid C_S(k^*_S, \alpha^*)] \leq E_P[\tilde{y} \mid C_P(k^*_S, \alpha^*)] \\
\alpha^* &\in (0, 1) \Rightarrow E_P[\tilde{y} \mid C_S(k^*_S, \alpha^*)] = E_P[\tilde{y} \mid C_P(k^*_S, \alpha^*)] \\
\alpha^* &= 1 \Rightarrow E_P[\tilde{y} \mid C_S(k^*_S, \alpha^*)] \geq E_P[\tilde{y} \mid C_P(k^*_S, \alpha^*)]. \tag{48}
\end{align*}
\]

Next we show that if

\[
\alpha < \frac{1 - \rho_S}{(1 - \rho_P) + (1 - \rho_L)} \Rightarrow E_P[\tilde{y} \mid C_S(k_S, \alpha)] > E_P[\tilde{y} \mid C_P(k^*_S, \alpha)]. \tag{49}
\]

Then from equations (49) and (48) we can conclude that optimal \(\alpha, \alpha^* \geq \frac{1 - \rho_S}{(1 - \rho_P) + (1 - \rho_L)}\). Thus, we need to establish (49). Let us first define the following functions:

\[
\begin{align*}
g^0(x, y, k_S, k_P, \alpha) &= N_0^0(k_S, k_P) + N_1^0 x + N_2^0(\alpha) y \\
N_0^0(k_S) &= (1 - \rho_P)((1 - \rho_S) k_S - \rho c) \\
N_1^0 &= \rho_S (1 - \rho_P) \\
N_2^0(\alpha) &= (1 - \alpha) \rho_P; \\
g^1(x, y, k_S, k_P, \alpha) &= N_0^1(k_S) + N_1^1 x + N_2^1(\alpha) y \\
N_0^1(k_S) &= (1 - \rho_P) k_S - (1 - \rho_S) (K - k_S) - (\rho_P - \rho_S) c + \rho_S \ell \\
N_1^1 &= \rho_P - \rho_S \\
N_2^1(\alpha) &= (1 - \rho_S) - \alpha ((1 - \rho_S) + (1 - \rho_P)).
\end{align*}
\]

Performing algebra similar to the algebra performed in the previous sections, we see that the states of the world over which parent and subsidiary debt restructuring are performed are given as follows:

\[
\begin{align*}
C_S(k_S, \alpha) &= \{(x, y) : g^0(x, y, k_S, \alpha) > 0\} \cap \{(x, y) : g^1(x, y, k_S, \alpha) > 0\}; \\
C_P(k_S, \alpha) &= \{(x, y) : g^0(x, y, k_S, \alpha) > 0\} \cap \{(x, y) : g^1(x, y, k_S, \alpha) \leq 0\}. \tag{50}
\end{align*}
\]

Next note that if \(\alpha < \frac{1 - \rho_S}{(1 - \rho_P) + (1 - \rho_L)}\), then the \(N_2\) coefficients in the definition of \(g^0\) and \(g^1\) are both positive. In this case, we see that the set \(C_S(k_S, \alpha)\) is an upper half plane lying above the set \(C_P(k_S, \alpha)\) in the \(x-y\) plane. Given our assumption of non-negative dependence between \(\tilde{x}\) and \(\tilde{y}\), the fact that the set \(C_S(k_S, \alpha)\) is an upper half plane lying above the set \(C_P(k_S, \alpha)\) in the \(x-y\) plane implies that the conditional expectation of \(\tilde{y}\) over \(C_S(k_S, \alpha)\) exceeds the expectation.
Thus, we have established (49). Note that both equations (48) and (49) imply the following result:

**Theorem 7.** Whenever the return on parent assets-in-place is non-negatively dependent on the return on the going concern value of the investment project located in the subsidiary, the firm will always pledge at least the fraction \( \frac{1-\rho_S}{(1-\rho_S)+(1-\rho_P)} \) of assets in place to increase the security of its subsidiary-level loans.

**Proof.** Follows directly from equation (48).

The intuition behind Theorem 7 is as follows: The firm’s objective in allocating assets in place is to minimize its own ex post ability to renegotiate contractual obligations. By assumption, creditor-group \( P \)'s claims are protected by a more creditor-friendly legal regime. For this reason, if all assets were also assigned to back creditor-group \( P \)'s claims, then the shareholder would opt for creditor-group \( P \)'s debt restructuring only in very adverse distress states. In these states, total assets in place would likely be small and thus of little benefit in strengthening creditor bargaining power. By guaranteeing subsidiary debt through the pledge of parent assets in place, the firm is able to ensure that assets in place work more effectively to strengthen the hand of creditors and thus lower overall agency costs. This effect is most pronounced when the subsidiary’s legal regime is unfriendly to creditors, in which case significant guarantees are required to redress the balance. In summary, our analysis predicts that the firm will always try to commit some assets to the subsidiary’s creditors, either through internal equity investment in the project or through loan guarantees. The minimum commitment is falling in the quality of the subsidiary’s legal regime. For this reason, our analysis predicts not only that less debt will be issued in weak (from a creditor’s perspective) legal systems, but also that such debt will feature more collaterals and guarantees.

### 8.2 Other extensions

It is possible to further extend our analysis. Some key factors to consider are (a) incentive conflicts between management at the subsidiary and parent companies; (b) asymmetric information between the firm and capital providers; and (c) tax rate differential between the subsidiary and parent. Because subsidiary management has considerable firm-specific capital tied up in operations in the subsidiary’s country, as well as a great deal of private information regarding the local prospects of the firm, incentive conflicts are possible. These conflicts can be mitigated by the financing package used by the subsidiary. For example, linking the level of subsidiary debt to local management reports of investment prospects in the local country could reduce local management’s incentives to overestimate the profitability of local investment so as to increase the level of investment and augment their firm-specific human capital.

Similarly, introducing external asymmetric information—asymmetric information between the firm and the subsidiary—might have a significant effect on our results. If management had private information on the likelihood of financial distress, the choice of the capital market used to obtain financing would be viewed by the financial market as a signal of value. In this case, minimizing ex ante agency costs would not be the only objective in initial security design; instead,

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23While this assertion is not surprising, the authors have not located a formal proof that non-negative dependence implies this assertion. For this reason, the authors have prepared a formal proof. It is available upon request.
the firm would balance agency costs against mispricing losses from asymmetric information. Adverse private information would provide an incentive for mimicry. If private information related only to the likelihood of financial distress, but not to the level of cash flows conditional on distress, our results would remain essentially unchanged because, as was shown in Section 4, optimal designs are always determined by the distribution of cash flows in the distressed state. However, if private information regarding the distribution of cash flows in financial distress were present, signal-jamming equilibria could arise, prompting all firms to mimic the capital structure choice associated with better distress-cash-flow distribution. Because superior distributions are associated with capital structures more evenly “balanced” between local and parent financing, asymmetric information, through signal jamming, might muffle the effects of legal regime differences on capital structure and lead to an increased diversity of funding sources. If the agency costs of a suboptimal mix between subsidiary and parent-country financing were large enough, separating equilibria might obtain in which managers signaled positive private information by increasing their reliance on subsidiary-county capital markets. The empirical prediction in this case would be that overseas borrowing is a positive signal.

Tax rate differentials would also have a significant impact if introduced into our analysis. With the introduction of differential taxation, the firm would aim not only to minimize the agency costs associated with debt issuance, but also maximize the value of the overall tax shield from debt. The specific effect of differential taxes on the analysis is more difficult to conjecture than the effects of external asymmetric information and internal incentive conflicts. The effect of tax rate differentials on the analysis will depend on how differentials in tax rates line up with differentials in creditor rights. In general, this alignment would seem to depend on the particulars of the legal systems of the subsidiary and parent countries.

9 Conclusion

In this paper, we developed a model of the financing policy of international corporations. The analysis showed that parent–subsidiary organizations will attempt to minimize the overall credit spread on their debt by equating the marginal enforceability of debt claims in different capital markets. The diminishing marginal enforceability of claims leads to internal optimal mixes between subsidiary borrowing and borrowing at the parent level. The exact mix of financing chosen depends on the creditor-friendliness of the parent and subsidiary countries, the going concern versus liquidation value of the firm, the total external financing needs of the firm, and the distribution of operating cash flows. Testable comparative statics are then derived which link capital structure, debt renegotiation strategy, security allocation, and the impact of currency risk, to these variables.

The model makes strong predictions. Are these predictions empirically relevant? Can they be tested? We believe that the model has significant advantages over most extant models in regard to testability. The advantage of our analysis in testability is that most of the results in this paper characterize the proportions of total debt financing provided or guaranteed in a given country. The advantage of examining these relationships is that a number of factors that affect the total level of debt financing or the level of overall assets in place (e.g., the marginal relationship between effort and the probability of financial distress) do not affect these optimal mix variables. Thus, by controlling for the overall level of debt financing and focusing on the
optimal mix, we effectively “screen” out a number of variables in the construction of the theory that we would otherwise be forced to control for econometrically in designing our tests. This should make it fairly simple to test our model. The dependent variables in such a test would be the fraction of financing obtained by the firm at the subsidiary (parent) level and perhaps the fraction of subsidiary debt guaranteed by the parent. The independent variables would be proxies for the legal systems in the different countries in which the firm operates. In addition, variables to control for the properties of the firm’s cash flows (e.g., industry dummies) would be required. Such a test would permit our legal systems hypotheses to compete directly with the heretofore dominant perspectives on financial management.

References


A1 The Explicit Bargaining Game that Produces the Renegotiation Outcomes

A bargaining game is played between the owners of the pso (henceforth called the “shareholder”) of and its creditors by backward induction. The game has the following simple structure. The manager approaches one of the two creditors. We call the creditor approached the first creditor. The manager either affirms or repudiates his contract with that creditor. In practice, “affirming” the debt will amount to little more than declining to renegotiate the debt contracts of the creditor. If contracts are affirmed, bargaining ends with the first creditor’s contracts in place.

If the manager repudiates the claim, he proposes a new claim structure to the creditor (either creditor-group \( S \) or creditor-group \( P \)). If the creditor accepts this offer, the claim structure is changed accordingly.

- In negotiations with creditor-group \( S \), claims on subsidiary cash flows (creditor-group \( S \)’s debt claim), \( k_S \), are the subject of negotiations.
- In negotiations with creditor-group \( P \), claims on the remitted cash flows (creditor-group \( P \)’s debt claim), \( k_P \), is the subject of negotiations.

If an initial offer by the manager is rejected, negotiations break down with probability \( 1 - \rho_j \), where \( j = S \) or \( P \). If negotiations break down, all agents receive their payoffs in the event of default:

- \( \pi_P \) for the creditor-group \( P \) and
- \( \pi_S \) for the creditor-group \( S \), and
because \( K > \ell \) (by assumptions (3) and (4)), the shareholders of the parent company get zero.

If negotiations do not break down, the creditor makes a final offer. The rejection of this offer triggers breakdown. This bargaining game is thus a two-move version of the Osborne and Rubinstein (1990) bargaining game. It is the simplest bargaining game that produces a non-trivial division of the surplus. Note that for any \( j = S \) or \( P \),

- if \( \rho_j = 1 \), then the creditor is able to make a final offer without risk of dissipation of value, and thus can capture all of the surplus from renegotiation.
- In contrast, if \( \rho_j = 0 \), rejection of the manager’s offer is suicidal for the creditor. Thus, the manager can capture all of the surplus.

Once negotiations with the first creditor end, the manager may never return to renegotiate claims against that creditor. The “never return” assumption is not required to obtain any of our results. As long as we follow the standard assumption of extensive bargaining games, that repeated rounds of negotiations are costly, we obtain exactly the same results. However, allowing for repeated returns to the same creditor in negotiations makes the analysis more cumbersome. For further analysis of this issue see Appendix A3.

Upon completing negotiations with one creditor, the manager turns to the next creditor. The manager again can either repudiate or affirm the claim of this creditor. Negotiations with the second creditor follow the same schema as negotiations with the first creditor. Once negotiations cease, the manager makes his ex post effort decision and the situation proceeds as in the timeline specified above.

The bargaining game determining the outcomes of debt renegotiation is a perfect information sequential game. Thus, unique subgame perfect equilibrium of the game can be identified by backward induction in a fairly transparent fashion. However, a complete characterization of the subgame perfect equilibrium of the game is not required for our subsequent analysis. All that we need is the division of value that occurs on the equilibrium path.

The equilibrium payoffs follow in straightforward fashion from the logic of the renegotiation problem. First note that \( k_S + k_P = K \geq I \). This result follows because creditors never receive more than the initial face value of their debt. Thus, in order to break even from providing financing, the face value of debt must at least equal the required investment, \( I \). Next, note that Assumption (C4) implies that, if some creditor’s claim is not renegotiated, ex post effort will not be applied and the manager will receive a zero payoff. Because the manager can do better than this by renegotiating, both or at least one claim must be renegotiated. Thus, the manager will repudiate either one of the claims or both of the claims. The payoff to the creditors is minimized (and thus the payoff to the manager is maximized) if the manager affirms one creditor’s debt before renegotiating with the creditor targeted for renegotiation. This result follows because affirmed payments to one creditor restrict the extractions of the other creditor. This reasoning shows that there are four candidate optimal negotiation strategies for the manager:

(a) Affirm creditor-group \( P \)’s claim, then negotiate creditor-group \( S \)’s claim;
(b) affirm creditor-group \( S \)’s claim, then renegotiate creditor-group \( P \)’s claim;
(c) first negotiate creditor-group \( S \)’s claim, then negotiate creditor-group \( P \)’s claim; and,
(d) first negotiate creditor-group \( P \)'s claim, then negotiate creditor-group \( S \)'s claim.

First consider the total payoffs to creditors if strategy (c) or (d) is followed. In both of these cases the total payments received by the creditors are the same and equal to

\[
\eta(\rho_S, \rho_P) \ell + (1 - \eta(\rho_S, \rho_P)) (x - c),
\]

(A1-1)

where \( \eta \) is defined by

\[
\eta(\rho_S, \rho_P) = (1 - \rho_S)(1 - \rho_P).
\]

(A1-2)

Let \( c' = S \) or \( P \) and \( c'' = S \) or \( P \neq c' \). Expression (A1-1) follows from noting that if the manager negotiates first with creditor \( c' \), then creditor \( c' \) can obtain \( x - c - \pi_{c''} \) if he rejects the manager’s initial offer, dissipation does not occur, and he makes a final offer for the entire surplus. Because dissipation occurs with probability \( 1 - \rho_{c'} \), the manager must offer at least this amount to the first creditor if the manager is to capture any of the value from the renegotiation. Because there is no incentive for the manager to offer more than this amount, the first creditor will receive

\[
\pi' = \rho_{c'}(x - c - \pi_{c''}) + (1 - \rho_{c'}) \pi_{c'}
\]

in any subgame perfect equilibrium. The first creditor’s claim must be satisfied in order for the manager to receive a positive payoff; thus, the first creditor’s claim is satisfied in any successful renegotiation. A final offer by the second creditor, then, must leave enough value both for the manager to receive a payoff of \( c \) and for the first creditor to be satisfied. Thus, the second creditor negotiated with will receive

\[
\pi'' = \rho_{c''}(x - c - \pi') + (1 - \rho_{c''}) \pi_{c'}.
\]

(A1-4)

Simplifying this expression, we note that \( \pi_{c'} + \pi_{c''} = \ell \) yields expression (A1-1) regardless of which creditor is chosen to be first for renegotiation.

Next, consider (a) and (b). If the manager follows strategy (a) by affirming creditor-group \( P \)'s claim and then renegotiating creditor-group \( S \)'s claim, his payoff will be positive only if creditor-group \( P \) is satisfied. Thus, whenever it is optimal for the manager to follow strategy (a), creditor-group \( P \)'s claim is satisfied; thus, total creditor payoffs under (a) are given by

\[
k_P + \rho_S (x - c - k_P) + (1 - \rho_S) \ell.
\]

(A1-5)

If the manager follows strategy (b) and affirms the subsidiary creditor’s debt while renegotiating the parent-country creditor’s debt, then it is again not possible for the parent company to receive a positive payoff if it defaults on subsidiary-country debt. Thus, the payoff from strategy (b) is

\[
k_S + \rho_P (x - c - k_S) + (1 - \rho_P) \pi_P.
\]

(A1-6)

We see from inspecting expressions (A1-5), (A1-6), and (A1-1), that these sequences may produce distinct creditor payoffs. Because the parent company picks the negotiating sequence, it will pick that sequence which minimizes the value of creditor claims. Thus, the payoff to creditors
is given by
\[
C^*(x, \rho_S, \rho_P, c, \ell, k_S, k_P) = \min \left[ \begin{array}{l}
(1 - \eta(\rho_S, \rho_P))(x - c) + \eta(\rho_S, \rho_P) \ell,

k_S (1 - \rho_P) + \rho_P (x - c) + (1 - \rho_P) \Pi_P,

k_P (1 - \rho_S) + \rho_S (x - c) + (1 - \rho_S) \ell
\end{array} \right].
\] (A1-7)

### A2 Proof that Maximizing Distress Cash Payments Maximizes the Ex ante Payoff of the Shareholder

To consider the initial debt decision, we first define the range of feasible initial financing policies, \((k_S, k_P)\). The space of admissible policies is given by
\[
C = \{ (k_S, k_P) : (k_S, k_P) \in [0, x(ND)] \}.
\] (A2-1)

The restriction of total debt to this region is without loss of generality because higher debt levels are never required to fund the project. Let \(s(k_S, k_P)\) represent the shortfall between the promised and realized payments after renegotiations conditioned on a given level of distress cash flows; that is,
\[
s(x, k_S, k_P) = k_S + k_P - C^*(x, k_S, k_P) = K - C^*(x, k_S, k_P).
\] (A2-2)

Let \(C(k_S, k_P)\) represent the expected value of creditor claims in the event of financial distress; that is,
\[
C(k_S, k_P) = \int C^*(x, k_S, k_P) G(dx).
\] (A2-3)

Let \(S\) represent the expected shortfall in the event of distress; that is,
\[
S(k_S, k_P) = (k_S + k_P) - C(k_S, k_P).
\] (A2-4)

In the above expression, \(S\) represents the expected extent of debt forgiveness when financial distress occurs. If the condition is nondistressed, then renegotiation of the debt claim is impossible, and the shareholders must pay off all debt claims, \(k_S + k_P\). Total value \(TV\) and the payoff to the shareholders are given by
\[
TV(e) = e \bar{x} + (1 - e) \left( \int x G(dx) - c \right) - \delta(e),
\]
\[
M(k_S, k_P, e) = TV(e) - (k_S + k_P) + (1 - e) S(k_S, k_P).
\]

The total value of the creditors’ claims is given by
\[
C(k_S, k_P, e) = k_S + k_P - (1 - e) S(k_S, k_P).
\] (A2-5)

Note that since the manager (and manager is aligned with shareholders) chooses the ex ante effort level \(e\) after the debt contracts are written, \(e\) is selected to maximize the equity holder’s payoff given \((k_S, k_P)\). When the manager issues debt, creditors must break even in expectation; that
is, the total value of their claims must equal the total capital they contribute, or, symbolically,

\[ C(k_S, k_P, e) = I. \]  \hfill (A2-6)

Note that there are two capital raising constraints—one for the subsidiary credit market and one for the parent credit market. However, since the manager is free to raise any or all of the capital needed to fund the project in either subsidiary or parent capital market, a financing policy satisfies these two constraints if and only if (A2-6) is satisfied. Using the capital raising constraint, (A2-6), we see that the contract design problem at date -2 can be expressed as follows:

\[
\begin{align*}
\max_{k_S, k_P} & \quad M(k_S, k_P, e) \\
\text{s.t.} & \quad C(k_S, k_P, e) = I, \\
& \quad e \in \text{Argmax}_{e \in [e, 1]} M(k_S, k_P, e) \\
& \quad (k_S, k_P) \in C.
\end{align*}
\]  \hfill (A2-7)

The first constraint in the above problem implies that this problem is equivalent to

\[
\begin{align*}
\max_{k_S, k_P} & \quad TV(e) - I \\
\text{s.t.} & \quad C(k_S, k_P, e) = I, \\
& \quad e \in \text{Argmax}_{e \in [e, 1]} M(k_S, k_P, e) \\
& \quad (k_S, k_P) \in C.
\end{align*}
\]  \hfill (A2-8)

Because \( TV \) is strictly increasing in \( e \), the above problem, (A2-8), has the same solution as the following one, which we call (A2-9):

\[
\begin{align*}
\max_{k_S, k_P} & \quad e \\
\text{s.t.} & \quad (i) \ C(k_S, k_P, e) = I, \\
& \quad (ii) \ e \in \text{Argmax}_{e \in [e, 1]} M(k_S, k_P, e), \\
& \quad (iii) \ (k_S, k_P) \in C.
\end{align*}
\]  \hfill (A2-9)

Thus, the optimal contract design problem (A2-9) amounts to choosing the contract design that maximizes ex ante effort subject to three constraints: capital raising (condition i), whereby the debt claim must allow creditors to break even given the manager’s choice of ex ante effort; incentive compatibility (condition ii), whereby the ex ante effort level chosen by the manager must be incentive compatible; contract design (condition iii), whereby the debt contract must lie in the feasible set \( C \).

Let \( e^* \) represent the incentive compatible level of ex ante effort; that is,

\[ e^* = \text{Argmax}_{e \in [e, 1]} M(k_S, k_P, e). \]  \hfill (A2-10)

The regularity conditions we have imposed on the cost of ex ante effort, \( c \), permit a simple characterization of ex ante effort levels that are incentive compatible (i.e., satisfying condition
ii). This characterization is provided below:

\[
\text{If } c - S(k_S, k_P) > 0 \Rightarrow \delta'(e^*) = c - S(k_S, k_P); \tag{A2-11}
\]

and

\[
c - S(k_S, k_P) \leq 0 \Rightarrow e^* = \underline{e}. \tag{A2-12}
\]

Together the two expressions above imply that

\[
e^*(S) = \delta'^{-1}(\max[c - S, 0]), \tag{A2-13}
\]

where \(\delta'^{-1}\) is the inverse function of the first derivative of \(\delta\). The first-best level of ex ante effort maximizes the total payoff, \(TV\). Thus, the first-best ex ante effort level is the solution to the following problem:

\[
\max_{e \in [\underline{e}, 1]} TV(e). \tag{A2-14}
\]

Let \(e^{fb}\) represent the solution to this maximization problem. The first-order condition for the first-best solution requires that \(\delta'(e^{fb}) = c\). That is,

\[
e^{fb} = \delta'^{-1}(c). \tag{A2-15}
\]

From assumptions (1) and (3), we see that the debt payments in the nondistressed condition, \(k_P + k_S\), are never smaller than payments in the distressed state \(C^*\). This fact implies that \(S \geq 0\). Comparing (A2-11) and (A2-12) with (A2-15), we see that

\[
e^* \leq e^{fb}. \tag{A2-16}
\]

In other words, the level of ex ante effort is always weakly less than the first-best level of ex ante effort. Moreover, (A2-11) and (A2-12) imply that \(e^*\) is decreasing in the shortfall, \(S\), and is strictly decreasing whenever \(S < c\). Thus, maximizing ex ante effort is equivalent to minimizing the extent of debt forgiveness when financial distress occurs.

**Lemma 5** (Technical Lemma A2.1). Let \((k_S^*, k_P^*)\) be a candidate optimal policy and let \(K = k_S^* + k_P^*\). Then if \((k_S^*, k_P^*)\) solves (A2-7) and the level of effort under the optimal policy exceeds the minimum level of effort, \(\underline{e}\), then it must be the case that \((k_S^*, k_P^*)\) solves

\[
\min_{k_S, k_P} S(k_S, k_P) \quad \text{s.t.} \quad (i) \ k_S + k_P \geq K \\
(ii) \ (k_S, k_P) \in C. \tag{A2-17}
\]

**Proof.** We shall show that if \((k_S^*, k_P^*)\) does not solve (A2-17), then it cannot solve (A2-7). To see this, first note that the failure of \((k_S^*, k_P^*)\) to solve (A2-17) implies that there exists \((k_S', k_P')\) such that \(k_S' + k_P' \geq K = k_S^* + k_P^*\) and \(S' \equiv S(k_S', k_P') < S(k_S^*, k_P^*) \equiv S^*\). At \(S^*\) the effort level exceeds \(\underline{e}\), which implies, as can be seen from inspecting (A2-13), that \(e^*\) is decreasing for
all \( S < S^* \). Consider the function

\[
\phi(\alpha) = S(\alpha k_S, \alpha k_P). \tag{A2-18}
\]

It is easy to see from equation (A2-4), that for every \( x \) the shortfall is increasing in \( \alpha \). This fact implies \( \phi \) is increasing in \( \alpha \). Let

\[
\alpha' = \min\{\alpha \in [0, 1] : g(\alpha) \geq I\},
\]

\[
g(\alpha) = C(\alpha k_S, \alpha k_P, e^*(\phi(\alpha))). \tag{A2-19}
\]

By our assumption that \( (k'_S, k'_P) \) produces a smaller shortfall than \( (k^*_S, k^*_P) \), and the fact that \( e^* \) is decreasing, it follows that the effort level under \( S' \) is higher. The sum of the nominal payments is the same under both contracts \( (k'_S, k'_P) \) and \( (k^*_S, k^*_P) \). Because the value of the creditor’s claim depends only on the sum of the nominal payments, \( K \), and the shortfall, it thus follows that \( C(k'_S, k'_P, e^*(\phi(1))) > C(k^*_S, k^*_P, e^*(S^*)) \). Because \( (k'_S, k'_P) \) solves (A2-7), \( C(k'_S, k'_P, e^*(S^*)) = I \). Thus, we have shown that \( g(1) > I \). Because, evaluated at 0, the \( g \) function determines total creditor payoffs from a contract promising no payments, we have that \( g(0) = 0 < I \). Because \( g(1) > I, g(0) < I, g \) is continuous, and \([0, 1]\) is compact, it follows that \( \alpha' < 1 \) and \( g(\alpha') = I \). Now, \( g(\alpha') = I \) implies that \( (\alpha k'_S, \alpha k'_P) \) is a feasible solution to (A2-7). Because \( \phi \) is increasing in \( \alpha \), equation (A2-13) implies that the equilibrium effort level associated with \((\alpha k'_S, \alpha k'_P)\) exceeds the equilibrium effort level associated with \((k^*_S, k^*_P)\). Because \((k'_S, k'_P)\) exceeds the equilibrium effort level and because \((\alpha k'_S, \alpha k'_P)\) satisfies the creditor breakeven condition for the optimal design problem (A2-7), it follows that \((k^*_S, k^*_P)\) is not an optimal policy. This contradiction establishes the lemma.

\[ \Box \]

### A3 Multiple Creditors and Claim Aggregation

Here we demonstrate that, assuming claim aggregation, multiple creditors in the same country will not be able to extract more from the parent-subsidiary management (henceforth, manager) than a single creditor. Thus, the assumption of a single creditor in each credit market is made with out any loss of generality and, moreover, multiple creditors in one credit market are not a substitute for other credit market claims. To show this, assume without loss of generality that the manager raises all financing in the subsidiary credit market and that liquidation value of assets is zero. We will show, in this setting, that multiple classes of subsidiary debt will not extract larger aggregate payments from the manager than a single class of debt under a broad class of claim aggregation schemata. The same arguments apply with mutatis minoribus mutandis to creditor-group \( P \)'s claims.

The schemata for institutional designs that produce this result is given below: The manager makes a first proposal in the first round (as in our basic model). If this proposal is accepted by all creditors then negotiations end; if the proposal is rejected by any creditor, then, as in the
basic model, there is a probability $1 - \rho$ of value dissipation. If value is not dissipated, then creditors make an offer. The proposal of creditors is determined in one of two ways:

(a) Nature randomly determines the creditor that will make the final offer. If the final offer is rejected, value is dissipated. Offers may be subject to absolute priority rules that block payout to junior creditors unless senior creditors are paid in full;

(b) an administrator maximizing an aggregate creditor welfare function that is strictly increasing in each creditor’s payoff makes the proposal, again perhaps subject to priority and/or security restrictions.

Although clearly the schemata listed above are not the only claim aggregation procedures conceivable, they are reasonable from an institutional perspective. For example, South Korean bankruptcy law allows large senior creditors to make the reorganization proposals (Economist, 1999). German bankruptcy law vests all counter-proposal authority in an administrator entrusted to maximize collective creditor payoffs (Economist, 1994). Other systems utilize creditor voting, which, assuming collective choice problems are resolved, resembles schema (b) above. Thus, the schemata we consider are consistent, at least with the ideals of claim resolution in most national bankruptcy systems.

The key to establishing the invariance of total creditor payoff to the number of creditors is to note that, regardless of the identity of the creditor making the final offer, or the priority structure of the firm’s debt, the payoff to any selected creditor, or an administrator maximizing the collective creditor payoff, is at least weakly increasing in total payoffs to creditors. Thus, an equilibrium will exist in which the final creditor proposal extracts the maximum possible surplus from the manager. This surplus is extracted by forcing the manager to make a payment of $x - c$ to creditors. Of course, depending on the identity of the creditor making the offer, the offers will vary. However, creditors will obtain in aggregate $x - c$. Averaging across all random allocations of the right to make a first offer, we can obtain an expected payoff for each creditor from the final offer. Call this expected payoff $\pi_j$, where $j$ indexes the creditors. These expected payoffs will sum to $x - c$. Because of the $1 - \rho$ probability of dissipation, each creditor $j$ will accept a payment of $\rho \pi_j$. Thus, to obtain consent required for the first stage, the manager must offer $\Sigma_j \pi_j \rho = \rho (x - c)$ in aggregate to all the creditors. This is exactly the same amount the manager must offer the single subsidiary creditor in our model. If, on the other hand, the manager does not opt for negotiation, he must satisfy the sum of the face values of the subsidiary creditors’ claims. Call this sum $k_S$. Since ex post the manager minimizes payments to creditors, his payments to creditors will equal $\text{Min}[k_S, \rho (x - c)]$. This is exactly the same payment as we obtained in our model for the case of a firm that relies only on subsidiary financing. Thus, assuming claim aggregation, multiple creditors in one country do not increase creditor payouts, and thus cannot ameliorate agency conflicts.
A4 Proofs of Selected Results

A4.1 Proof of Lemma 2

Substituting into problem $SP_F(K \mid \rho)$ the constraint that $k_S + k_P = K$ yields the following simplified problem:

$$\max_{k_S} C^*(x^o, k_S, K - k_S)$$

s.t $k_S \in [0, K].$ (A4-1)

First we consider the optimal policy when $k_S \leq \ell$. If the subsidiary debt level is less than $\ell$, the subsidiary debt contract will never be renegotiated. This implies that the parent debt contract has to be renegotiated. Thus, the payoff to creditors will equal

$$(1 - \rho) k_S + \rho_P (x^o - c) + (1 - \rho) \Pi_P = (1 - \rho) k_S + \rho (x^o - c) + (1 - \rho) (\ell - k_S).$$ (A4-2)

This expression is constant in $k_S$ and equal to

$$\rho (x^o - c) + (1 - \rho) \ell.$$ (A4-3)

In contrast, for all levels of subsidiary debt slightly larger than $\ell$ but still small enough so that restructuring only for creditor-group $P$ is optimal, we have that the payment to creditors is given by

$$(1 - \rho) k_S + \rho (x^o - c).$$ (A4-4)

Because $k_S > \ell$, expression (A4-4) is larger than (A4-3). Thus, we have shown that policies that involve subsidiary debt levels of less than $\ell$ do not maximize total creditor payoffs and thus do not solve problem $SP_F(K \mid \rho)$. Next consider policies featuring subsidiary debt levels in excess of $\ell$. If $k_S > \ell$, and $k_S + k_P = K$, then the total creditor payments, assuming parent debt is restructured, which we represent by $\text{PR}$, and subsidiary restructuring, which we represent by $\text{SR}$, are given as follows:

$$\text{PR}(k_S \mid \rho) = (1 - \rho) k_S + \rho (x^o - c),$$ (A4-5)

$$\text{SR}(k_S \mid \rho) = (1 - \rho) (K - k_S) + \rho (x^o - c) + (1 - \rho) \ell.$$ (A4-6)

As shown in (A1-1), the payoff from renegotiating both claims, which we represent by $\text{A}$, is given by

$$\text{A} = \eta(\rho) \ell + (1 - \eta(\rho)) (x^o - c).$$ (A4-7)

Thus, the objective function $C^*$ takes the following form:

$$C^*(x, k_S, K - k_S) = \min[\text{A}, f(k_S)],$$

$$f(k_S) = \min[\text{SR}(k_S \mid \rho), \text{PR}(k_S \mid \rho)].$$ (A4-8)

The above observation implies that

$$\max_{k_S \in[0, K]} SP_F(K \mid \rho) = \max_{k_S} \min[\text{A}, f(k_S)].$$ (A4-9)
Next, note that the maximum of $f$ over the feasible region must be attained at a point where, $SR(k_S | \rho) = PR(k_S | \rho)$. Otherwise, by pushing up the smaller component the $f$ can be increased. Thus, $f$ is maximized at

$$(1 - \rho)k_S + \rho(x^o - c) = (1 - \rho)(K - k_S) + \rho(x^o - c) + (1 - \rho)\ell. \quad (A4-10)$$

The solution to (A4-10) is given by

$$k_S^* = \frac{K + \ell}{2} \text{ and } k_P^* = K - k_S^*. \quad (A4-11)$$

Because $A$ is a constant independent of the choice variables for problem $SP_F(K | \rho)$, we have that

$$k_S^* \in \text{Argmax}(SP_F(K | \rho)). \quad (A4-12)$$

Let $f^*$ equal the maximum of $f$, then a simple calculation shows that

$$f^* = \max_{k_S \in [0,K]} f(k_S) = f(k_S^*) = \rho \left( x^o - c \right) + (1 - \rho) \left( \frac{1}{2}(K + \ell) \right). \quad (A4-13)$$

Equations (A4-9), (A4-12) and (A4-13) imply that

$$\max SP_F(K | \rho) = \min \left[ A, \rho \left( x^o - c \right) + (1 - \rho) \left( \frac{1}{2}(K + \ell) \right) \right]. \quad (A4-14)$$

Hence, the proof. ■

### A4.2 Proof of Theorem 1

Using the expression (A4-14) it is straightforward to show that

$$f^* = \min[f] > A \quad (A4-15)$$

holds if and only if

$$\rho < \rho^* = \frac{1}{2} \frac{(K - \ell)}{x - c - \ell} \quad (A4-16)$$

Optimal polices $k_S^*$ satisfy the condition that

$$\min[f(k_S^*), A] = \min[f^*, A] \quad (A4-17)$$

(recall the definition of the function $f$ given by (A4-7)). Thus, when (A4-16) holds, optimal policies are all those policies $k_S^*$ satisfying the condition that $f(k_S^*) \geq A$. Algebra shows that these are the policies given by Theorem 1. This result establishes part (i) of Theorem 1.

Next, consider part (ii). Of Theorem 1 If (A4-15) does not hold, then (A4-16) is not satisfied. In this case, we have that $f^* = \max f \leq A$. Optimal policies $k_S^*$ satisfy the condition that $\min[f(k_S), A] = \min[f^*, A]$. Thus, when condition (A4-16) fails, optimal policies are all those policies $k_S^*$ satisfying the condition that $f(k_S) = f^*$. By the proof of Lemma 4, solutions to this problem are given by $(k_S^*, k_P^*)$ as defined in Theorem 1. ■
A4.3 Proof of Theorem 2

To prove condition (1), simply inspect the explicit solutions given in Theorem 1. Note that all solutions involve interior levels of subsidiary financial policies. As \( \rho \) increases, \( k_S(\rho) \) and \( k_S^+ (\rho) \) both decrease. Hence, any \( \lambda k_S(\rho) + (1 - \lambda) k_S^+ (\rho) \) also decreases. Since \( k_P(\rho) = K - k_S(\rho) \), the opposite comparative statics hold for \( k_P \). From Lemma 2 we know \( k^*_S \) is independent of \( \rho \) and remains unchanged. (Similar arguments establish the comparative static for going-concern value, that is, condition (3)). To prove condition (4), note that when \( k_S \leq \ell \),

\[
C^* = \rho(x - c) + (1 - \rho) \ell. \tag{A4-18}
\]

Next note that inspection of (17) shows that there exists \( k_S \) such that \( k_S + k_P = K \) and \( k_S > \ell \).

\[
C^* = \rho(x - c) + (1 - \rho) k_S. \tag{A4-19}
\]

Comparing (A4-18) with (A4-19) shows that payments to creditors are higher for some \( k_S > \ell \) than they are at any \( k_S \leq \ell \). This establishes the result. \( \blacksquare \)

A4.4 Proof of Lemma 4

Substituting into \( SD(K) \) the constraint that \( k_S + k_P = K \) yields the following simplified problem.

\[
\max_{k_S} C^*(x^0, k_S, K - k_S)
\]

s.t \( k_S \in [0, K] \). \( \tag{A4-20} \)

First we consider the optimal policy when \( k_S \leq \ell \). If the subsidiary debt level is less than \( \ell \), the subsidiary debt contract will never be renegotiated. This implies that the parent debt contract has to be renegotiated. Thus, the payoff to creditors will equal

\[
(1 - \rho_P) k_S + \rho_P (x^0 - c) + (1 - \rho_P) \Phi = (1 - \rho_P) k_S + \rho_P (x^0 - c) + (1 - \rho_P) (\ell - k_S). \tag{A4-21}
\]

This expression is constant in \( k_S \) and equal to

\[
\rho_P (x^0 - c) + (1 - \rho_P) \ell. \tag{A4-22}
\]

In contrast, for all levels of subsidiary debt slightly larger than \( \ell \) but still small enough so that restructuring only creditor-group \( P \) is optimal, we have that the payment to creditors is given by

\[
(1 - \rho_P) k_S + \rho_P (x^0 - c). \tag{A4-23}
\]

Because \( k_S > \ell \), expression (A4-23) is larger than (A4-22). Thus, we have shown that policies that involve subsidiary debt levels of less than \( \ell \) do not maximize total creditor payoffs and thus do not solve \((SD(K))\). Next consider policies featuring subsidiary debt levels in excess of \( \ell \). If \( k_S > \ell \), and \( k_S + k_P = K \), then the total creditor payments, assuming parent debt is restructured, which we represent by \( PR \), and subsidiary restructuring, which we represent by
SR, are given as follows:

\[
\begin{align*}
PR(k_S) &= (1 - \rho_P)k_S + \rho_P(x^o - c), \\
SR(k_S) &= (1 - \rho_S)(K - k_S) + \rho_S(x^o - c) + (1 - \rho_S)\ell.
\end{align*}
\]

(A4-24) (A4-25)

As shown in (A1-1), the payoff from renegotiating both claims, which we represent by \(A\), is given by

\[
A = \eta(\rho_S, \rho_P)\ell + \left(1 - \eta(\rho_S, \rho_P)\right)(x^o - c).
\]

(A4-26)

Thus, the objective function \(C^*\) takes the following form:

\[
C^*(x, k_S, K - k_S) = \min[A, f(k_S)],
\]

\[
f(k_S) = \min[SR(k_S), PR(k_S)].
\]

(A4-27)

The above observation implies that

\[
\max SD(K) = \max_{k_S \in [0, K]} \min[A, f(k_S)].
\]

(A4-28)

Next, note that the maximum of \(f\) over the feasible region must be attained at a point where, \(SR(k_S) = PR(k_S)\). Otherwise, by pushing up the smaller component the \(f\) can be increased. Thus, \(f\) is maximized at

\[
(1 - \rho_P)k_S + \rho_P(x^o - c) = (1 - \rho_S)(K - k_S) + \rho_S(x^o - c) + (1 - \rho_S)\ell.
\]

(A4-29)

The solution to (A4-29) is given by

\[
k_{S}^* = \frac{(1 - \rho_S)K + (\rho_S - \rho_P)(x - c) + (1 - \rho_S)\ell}{(1 - \rho_S) + (1 - \rho_P)}.
\]

(A4-30)

Because \(A\) is a constant independent of the choice variables for problem SD(K), we have that

\[
k_{S}^* \in \text{Argmax}(SP_F(K | \rho)).
\]

(A4-31)

Let \(f^*\) equal the maximum of \(f\), then a simple calculation shows that

\[
f^* = \max_{k_S \in [0, K]} f(k_S) = f(k_{S}^*) = (1 - \phi^*)(x^o - c) + \phi^*\left(\frac{1}{2}(K + \ell)\right),
\]

(A4-32) where

\[
\phi^* = \frac{(1 - \rho_S)(1 - \rho_P)}{\frac{1}{2}\left((1 + \rho_S) + (1 + \rho_P)\right)}.
\]

(A4-33)

Equations (A4-32) and (A4-33) imply that

\[
\text{Max}(SD(K)) = \min\left[A, (1 - \phi^*)(x^o - c) + \phi^*\left(\frac{1}{2}(K + \ell)\right)\right].
\]

(A4-34)

Hence, the proof. ■
A4.5 Proof of Theorem 3

A simple calculation shows that

$$\bar{\rho} \frac{1}{1 - \bar{\rho}} < \frac{1}{2}(K - \ell)$$

(A4-35)

holds, if and only if

$$f^* = \min[f] > A.$$  (A4-36)

Optimal policies $k_S^*$ satisfy the condition that

$$\min[f(k_S), A] = \min[f^*, A]$$

(A4-37)

(recall the definition of the function $f$ given by (A4-27)). Thus, when (A4-35) holds, optimal policies are all those policies $k_S^*$ satisfying the condition that $f(k_S^*) \geq A$. Algebra shows that these are the policies given by Theorem 4. This result establishes part (i) of Theorem 4.

Next, consider part (ii) of Theorem 3. If (A4-35) does not hold, then (A4-36) is not satisfied. In this case, we have that $f^* = \max f \leq A$. Optimal policies $k_S^*$ satisfy the condition that $\min[f(k_S), A] = \min[f^*, A]$. Thus, when condition (A4-36) fails, optimal policies are all those policies $k_S^*$ satisfying the condition that $f(k_S) = f^*$. By the proof of Lemma 4, solutions to this problem are given by $(k_S^*, k_P^*)$ as defined in the theorem. ■

A4.6 Proof of Theorem 4

To prove part (i), simply inspect the explicit solutions given in Theorem 4. Note that all solutions involve interior levels of subsidiary creditor financial policies. To prove part (ii), note that the function $h(k_P, \rho_S, \rho_P) := C^*(x^0, K - k_P, k_P, \rho_S, \rho_P)$ exhibits decreasing differences in $\rho_S$ and increasing differences in $\rho_P$; this implies that the set of optimizers of the function are decreasing in $\rho_S$ and increasing in $\rho_P$ in the induced set ordering (see, e.g., Lemma 2.8.1, Topkis, 1998). Since $k_S = K - k_P$, the opposite comparative statics hold for $k_S$. Similar arguments establish the comparative static for going-concern value, that is, condition (3). This establishes the result. ■

A4.7 Proof of Theorem 5

Substitute into the problem the constraint that $K = k_S + k_P$ and note that the function $C^*$ is almost everywhere differentiable in $x$ and that $G$ is absolutely continuous, with a derivative in $k_S$ of $(1 - \rho_P)$ over $[\hat{a}, \hat{b}]$ and $-(1 - \rho_S)$ over $[\hat{b}, \bar{x}]$ (or use Leibniz’s rule to explicitly differentiate $C$). We see that the first-order condition for an optimal choice of $k_S$ is given by

$$\frac{dC}{dk_S} = (1 - \rho_P)\left(G(\hat{b}(k_S, K - k_S)) - G(\hat{a}(k_S, K - k_S))\right) - (1 - \rho_S)\left(1 - G(\hat{b}(k_S, K - k_S))\right) = 0.$$  (A4-38)

At $k_S = 0$ the derivative is positive and at $k_S = K$ the derivative is negative; because the derivative is continuous, the first-order condition (A4-38) must be satisfied at any maximum. Because $C^*$ is concave in the control variable $k_S$ for all $x$, and because expectations preserve concavity, $C$ is concave. Thus, condition (A4-38) is sufficient for an optimum. Hence, the proof. ■
A4.8 Proof of Theorem 6

Let us denote the strategy of repudiating creditor-group $S$’s claim and affirming creditor-group $P$’s claim as $S_1$ and the strategy of repudiating creditor-group $P$’s claim and affirming creditor-group $S$’s claim as $S_2$.

If $d > d^*$, then we know from equation (40) that

$$\frac{d(1 - \rho_P) + (\rho_P - \rho_S)(1 - \rho_S)(K + c - x)}{2 - \rho_P - \rho_S} > 0,$$

implying that the debt repayment in the distress state from adopting strategy $S_2$, $C^*(S_2)$, is greater than $C^*(S_1)$, the debt repayment in the distress state if the shareholder adopts strategy $S_1$. But we also know that the shareholder chooses initial borrowing at the subsidiary level, $k^d_S$, and the parent level, $k^d_P$, such that shareholder’s payoff from adopting strategy $S_1$ is the same as shareholder’s payoff from adopting strategy $S_2$.

Now, suppose the manager finds it optimal to negotiate with creditor-group $S$ rather than creditor-group $P$; that is, adopt strategy $S_1$ rather than $S_2$. Given that the risk free rate is zero and capital markets are competitive, we know that

$$e^*(S_1) K^*(S_1) + (1 - e^*(S_1)) C^*(S_1) = I,$$  \hspace{1cm} (A4-40)

where $e^*(S_1)$ is the optimal effort conditional on the manager’s adopting $S_1$ in the distress state and $K^*(S_1)$, equal to $k^*_S(S_1) + k^*_P(S_1)$, is the face value of the aggregate debt. Hence,

$$e^*(S_1) K^*(S_1) + (1 - e^*(S_1)) C^*(S_2) > I.$$  \hspace{1cm} (A4-41)

Suppose the manager reduces $K^*(S_1)$ and keeps $e^*(S_1)$ constant. Then, $(1 - e^*(S_1)) C^*(S_2)$ remains constant too. Given these are continuous functions, a drop in $K^*(S_1)$ will make equation (A4-41) hold with strict equality. This means, although the disutility of effort remains the same as the effort choice remains the same, the shareholders payoff in the nondistress state, $X - K$, goes up. Hence, the shareholders are strictly better off. Hence, $S_1$ cannot be the optimal strategy.