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A Welfare Analysis of Hot Money

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Abstract

There is ample evidence that cross-country contagion of financial crisis is caused by short-term capital flows (hot money). It is sometimes used in order to justify a policy of restricting the integration of the world's liquidity markets (fragmentation). In a model with under-provision of liquidity, contagion and excessively-high probability of financial crisis, we show that fragmentation has an ambiguous effect on welfare. Indeed, we show that when a country encloses liquidity into its own market it deprives its neighbour of liquidity, increasing the neighbour's probability of financial crisis. Fragmentation may also imply that a country may "sit" on idle liquidity while its neighbour suffers from a financial crisis. Hence, there are conceivable circumstances where the welfare cost of fragmentation dominate the benefits, giving rise to "optimal contagion" in the second-best sense. Policy coordination plays an important role in our analysis.

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1. INTRODUCTION

The Asian Crisis convinced many that short-term capital flows, *hot money*, are potentially damaging and, therefore, should be restricted. According to Bhagwati (1998) there is "striking evidence of the inherently crisis-prone nature of free capital movements". Calvo (1998) coined the term "sudden stop" to describe a causal chain whereby an outflow of hot money towards a liquidity-short foreign country can trigger a domestic financial crisis by way of *contagion*. More recently, Stiglitz (2010) argued that liquidity crises "associated with forced sales of asset ... [provides] a compelling reason that global integration may not be desirable". In this paper we accept that contagion should play a central role in the analysis of hot money. At the same time we question whether it justifies a policy of *fragmentation* (namely, a reversal of integration).

The reason for our skepticism is that the incidence of observed contagion needs to be evaluated against the counterfactual of crises that could have materialized but have been avoided due to the inflow of capital from abroad. Hence, a trade-off may exist where more contagion in some circumstances should be tolerated for a better chance of avoiding crisis in other circumstances, giving rise to *optimal contagion* (in a second-best sense). To put it differently, by enclosing liquidity into its own market, a country may deprive its neighbor from liquidity but, absent policy *coordination*, fail to internalize the welfare implications. Hence, the old *beggar-thy-neighbor* effect, whereby a country attempts to resolve domestic economic problems at the expense of its neighbor (Stiglitz, 1999) should also play a central role in the analysis. The main question is, thus, whether a properly modelled contagion effect is strong enough to dominate any other consideration. We answer the question through exact welfare accounting of hot money.

To that end, we construct a two-country model where speculators invest in liquidity in order to profit from low fire-sale prices, giving rise to a cash-in-the-market effect (as in Allen and Gale, 1998). Fire-sales of investment goods are a direct consequence of secured lending as in Hart and Moore (1998). Though contracts are optimally negotiated between debtors and creditors to extract all private gains from trade (given market conditions), bids in the fire-sale market fail to reflect the entire social value of repossessed collateral. Hence, competitive markets are grossly inefficient, liquidity is a public good and, as such, suffers an under-provision problem. Moreover, the competitive equilibrium captures many stylized facts related to financial crisis: it occurs with excessively high (strictly positive) probability and is characterized by a sharp increase in repossessions, fire sales, a drop in investment-goods prices and credit rationing (a credit crunch). We abstract the analysis from technological shocks; all projects are economically viable, though some may suffer from financial distress due to a temporary shortage of income. The macro shock is modeled as a pure redistribution of wealth *within* each economy, from capital-poor to capital-rich entrepreneurs. The purpose of this abstraction is twofold: to account for absent evidence that links financial crisis to any "real" factor, and to focus the analysis on the pivotal role of liquidity.

Three results deserve special attention. First, we characterize a *triage rule*[1], whereby countries commit ex-ante to an allocation mechanism, contingent on the realization of their liquidity shock. It turns out that, in some cases, it is ex-ante Pareto-optimal to infect one country with a mild financial crisis in order to save the other from a more serious one. Clearly, by itself, contagion is not (constrained) sub-optimal. Second, we explore the complex externalities generated by unilateral fragmentation policies where the government of each country encloses liquidity into its own market but ignores the effect of the policy on its neighbor. Not only that liquidity can stand idle in one country while the other suffers from financial crisis, the unilateral implementation of fragmentation in one country *crowds out* hot money and, thus, decreases the amount of liquidity available to its neighbor. We show that each country has the incentive to employ such beggar-thy-neighbor policies up to the feasible maximum. Third, we show that when policies are coordinated and cross-country externalities internalized, fragmentation can be optimal only in some cases. Since the uncoordinated equilibrium is at full fragmentation, it follows that there is "too much fragmentation" in an uncoordinated equilibrium.

To sum up, while our model provides strong grounds to believe that liquidity is under-provided in a competitive equilibrium, it provides only limited grounds to believe that a policy of fragmentation contributes to ameliorating the problem. Moreover, policy co-ordination can mitigate the externalities associated with fragmentation and contribute to social welfare. At a minimum, coordination should moderate the tendency towards excessive fragmentation in an uncoordinated equilibrium. Even better, an IMF-like institution can be created, which would hold liquidity and dispatch it conditional on the realized shocks in the two countries, resolving the commitment problems inherent in the triage rule.[2]

Though we frame our research question in the context of international finance, the analysis contributes to an understanding of a broader question: the role of markets in providing and allocating liquidity. For example, the question whether banks should be self-sufficient in reserve liquidity or rely on liquid markets to provide for shortages is related in some important respects to the one that we investigate here; see also Gale and Yorulmazer (2011). Our results seem to indicate that black-and-white, pro or anti market formulae are overly simplistic. For the market clearly fails to provide sufficient liquidity but, at the same time, the effect of crude quantitative restrictions is ambiguous, at best.

The paper is organized as follows: Section 2 presents the model, Section 3 analyzes the contract, and Section 4 develops a single-country benchmark. Section 5 presents the

[1] The concept is taken from emergency medicine where it provides priority rules for treating patients in trauma subject to scarce medical resources.

[2] Notice that the triage is ex-ante efficient, but ex-post a mildly infected country loses when its liquidity is used in order to save a more severely infected country.

two-country competitive equilibrium and Section 6 provides the welfare analysis. Section 7 concludes.

Literature review

Fisher (1933) provides the first academic analysis of the interrelations between market prices, leverage and economic activity. The first formal modelling is by Bernanke and Gertler (1989) and by Shleifer and Vishny (1992) who emphasize the fire-sale channel, followed by Kiyotaki and Moore (1997), Suarez and Sussman, (1997) and many others.[3] Allen and Gale, (1998) provide a link to the Diamond-Dybvig (1983) analysis of liquidity. The central source of inefficiency is the pecuniary externality inflicted by liquidation decisions, as in Bhattacharya and Gale (1987). Most of the building blocks that we use in the construction of our single-country benchmark already appear in these papers. We therefore focus below on contributions that are directly related to hot money, capital flow and, therefore, to our own work.

Caballero and Krishnamurthy (2001, 2004) are among the first to extend the analysis internationally. They model a small open economy where some assets are pledgeable as collateral to domestic lenders but not to foreigners. A fraction of the domestic firms will face financial distress in the interim period and will require additional funding. Though the economy is entirely dependent on funding from abroad, there is a domestic loan market where non-distressed firms borrow abroad and lend domestically utilizing their advantage in handling domestic collateral. In this kind of a setting, the amount of debt capacity that is left un-utilized ex ante in order to service interim distressed lending can be interpreted as liquidity provision. It is shown that the competitive equilibrium is inefficient. Moreover, an ex-ante borrowing tax can restore optimality by leaving more capacity available for the interim market.

Mendoza (2010) and Korinek (2011a) construct DSGE models augmented by a constraint that limits borrowing to a fraction of the *market value* of the collateral. The constraint is not derived from an explicit agency problem, yet it tightens endogenously in response to a change market conditions. The setting is simple enough to allow for a simulation of the model's rich dynamics. A negative productivity shock might generate a "feedback loop", leading to weak demand, low investment-goods prices and, thus, a reduction in borrowing capacity. The papers differ in focus. The first emphasizes that equilibrium dynamics differ materially from that of an open economy with a frictionless capital market during the (rare) events when the borrowing constraint binds. Perhaps the most dramatic "reversal" is in the dynamics of the trade balance: instead of creating a deficit so as to smooth out the effect of a negative productivity shock, the economy goes into a surplus that amplifies the effect of the shock. Korinek (2011a) has an ex-

[3] The following is a very incomplete list of some related recent contributions: Brunnermeier and Pedersen (2008), Bolton, Santos and Scheinkman (2009), Diamond and Rajan (2011), Goodhart et. al. (2012), Gorton and Huang (2004), Holmström and Tirole (1998, 2011), Lorenzoni (2008), Suarez and Sussman (2007).

plicit welfare analysis of an exchange economy. In a competitive equilibrium, consumers take capital-goods prices as given and, thus, fail to internalize the effect of their own borrowing on the likelihood that the constraint may tighten. In contrast, a planner can internalize the effect and tax the inflow of hot money. The optimal policy is to increase the capital-import tax by 0.87% when capital imports increases by 1%,

The paper by Acharya et. al. (2010) is, in some respects, the closest to ours. Liquidity is provided by specialized agents (called "arbitrageurs") who profit from buying fire sales assets. The amount of liquidity is determined endogenously. The key result is that the competitive amount of liquidity is economically inefficient. The model is then extended to the case of two countries. The analysis clearly identifies the possibility of contagion across countries and provides a condition under which the equilibrium amount of liquidity (per country) would increase as a result of a liquidity-market integration. The paper does not provide an explicit welfare analysis of the two-country case.

Our paper differs from the above contributions in several respects. We model in greater detail the underlying agency problem. Although this complicates the analysis, we believe it is worthwhile: first, because the interaction of incentives and market prices is at the core of financial-crisis analysis. Second, the technical difficulties generated by the structural assumptions, e.g. that our social-welfare problem has a strong tendency to yield a corner solution, may be a fundamental property of crisis models. None of the papers above has an explicit social-welfare analysis of the two-country case. Particularly, to the best of our knowledge, there is no analysis of the interaction between the beggar-thy-neighbor externality and contagion.[4] As a result, there is no clear distinction in the literature between the "pure" effect of fragmentation and its indirect effect on ameliorating the under-provision of liquidity problem.

2. THE MODEL

Consider a world with four periods $t = 0, ..., 3$. There are two countries, $i = A, B$, each with a measure-one continuum of entrepreneurs, who are the object of our welfare accounting. There are also some speculators who reside out of these countries and are excluded from the welfare accounting. All agents are risk neutral and maximize total $t = 3$ consumption (there is no discounting).

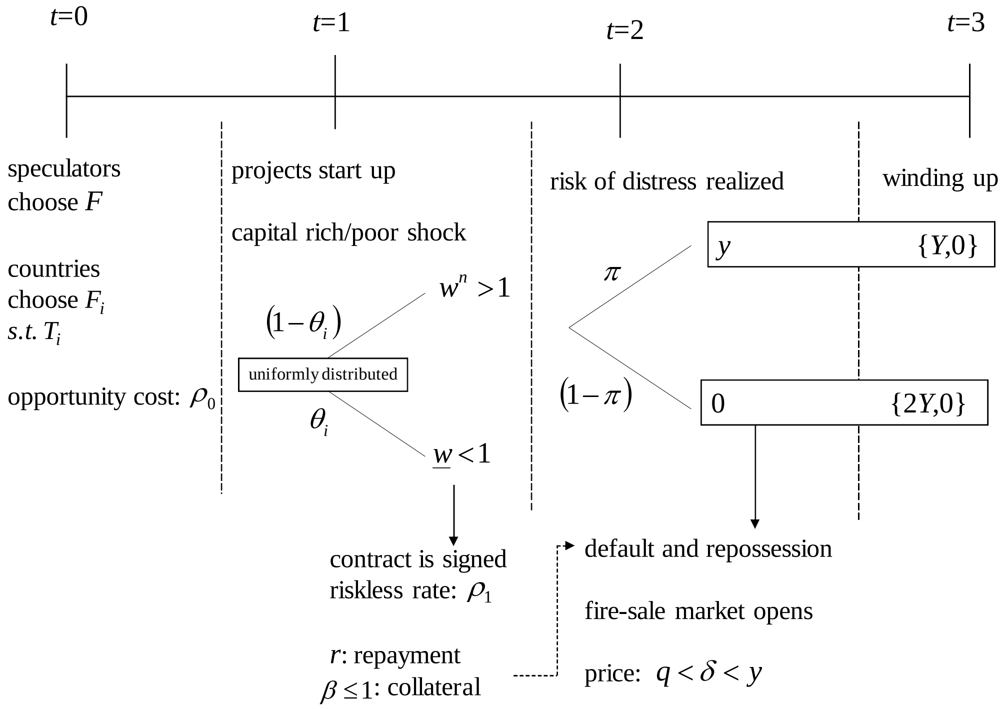
Speculators make their decisions at $t = 0$. Denote by $F \geq 0$ the amount of liquidity that they provide, jointly. Liquidity is held in the form of consumption goods, which are storable at a zero rate of return. Short positions are ruled out by the absence of $t = 0$ counter parties. At $t = 0$ (and then only) there is an alternative investment: an illiquid asset that yields a riskless gross return, $\rho_0 \geq 1$, at $t = 3$. Speculators have sufficient

[4] See, however, Korinek (2011b) where overborrowing in one country can be mitigated by restricting its capital *inflows*.

resources to satisfy any demand for liquidity (subject to profitability considerations). It follows that the supply of liquidity is perfectly elastic at $t = 0$ and perfectly inelastic thereafter. We shall interpret the free flow of liquidity across countries as hot money.

Figure 1

Time line



As we shall see, there is an under-provision of liquidity in a competitive equilibrium, which gives governments a motive to provide some extra liquidity, F_i. To do so, they borrow at market rate ρ_0 at $t = 0$. At $t = 3$ the governments levy taxes and pay back the debt. Tax distortions are convex in revenue. For simplicity we assume that there are no tax distortions up to a point T_i, so that the marginal cost of government liquidity up to that point is ρ_0. Beyond T_i the cost of tax distortions increases to infinity. The amount of government-supplied liquidity is common knowledge, so that the speculators can adjust the amount of liquidity that they provide to changes in the amount that the governments provide.

At $t = 1$ each entrepreneur gains access to one indivisible project, which requires an investment of one unit of a consumption good in order to start up. Each entrepreneur also receives an endowment of consumption goods, his capital, which he uses in order to fund the project. As for our macro shock, in each country there is a random fraction, θ_i, of capital-poor entrepreneurs with a $\underline{w} < 1$ endowment, which is fixed and independent of θ_i. All other entrepreneurs have an endowment of w_i^n, a random variable. On aggregate, each country has a non-random endowment capital w_a,

$$w_a \equiv \theta_i \underline{w} + (1 - \theta_i) w_i^n = 1. \tag{1}$$

It follows that for any realization of θ_i, $w_i^n \geq 1$, which makes the the capital-rich self sufficient in funding. Hence, the θ shock is purely re-distributive with no technological or aggregate-wealth implications. It does, however, create a strong motive for trading funds at $t = 1$ as the capital-poor seek funding from the capital rich. We do not model the redistributive shock but we have in mind sectorial capital losses due to, say, reckless trading or faulty risk management. These are recognized to play a major role in triggering a financial crisis. The incidence of poor capital is independent across the two countries and is uniformly distributed on $\left[0, \overline{\theta}\right] \times \left[0, \overline{\theta}\right]$. We assume that entrepreneurs cannot insure against being capital poor.[5]

Financial frictions aside, all projects are economically viable: *if* carried to maturity each project generates the same expected income, $2y$, $y > \rho_0$, of consumption goods. There is, however, a risk of financial distress due to a temporary shortage of income. The non-distress outcome occurs with a probability π, where the project generates an income y at $t = 2$; at $t = 3$ it generates a random income in $\{Y, 0\}$ with an expected value of y. The distress outcome occurs with a probability $1 - \pi$, where the project generates zero income at $t = 2$; at $t = 3$ it generates a random income in $\{2Y, 0\}$ with an expected value of $2y$. Financial distress is idiosyncratic and independent of θ_i.

After one period that the consumption good is invested in the project, it is transformed into an investment good. At that point ($t = 2$) a market is opened and a spot-price, q_i, is established. We also assume that once the investment good is detached from the investing entrepreneur, it can generate only a fraction of $\{Y, 0\}$ at $t = 3$, with an expected value $\delta < y$.[6] That repossession destroys value plays a central role in our welfare results. Clearly, q_i cannot exceed δ: $q_i \leq \delta$. Notice that the $t = 2$ investment-good price, q_i, can be perfectly anticipated at $t = 1$ when the macro shock, θ_i, is realized and observed. Potential lenders might prefer to hoard liquidity in order to buy investment goods rather than fund investment. Hence, an arbitrage relationship is established between the risk-free lending rate, $\rho_{1,i}$, and the price of investment goods:

$$\rho_{1,i} = \frac{\delta}{q_i}. \tag{2}$$

Income flows are private information to the entrepreneur who owns the project, and as a result non-pledgeable. By that we mean that one cannot write enforceable contracts contingent on income flows. In contrast, once consumption goods have been transformed into investment goods (we have in mind buildings or machinery) they become pledgeable. As we shall see, under the optimal contract a fraction β_i of the investment is pledged as collateral, which the lender has the right to repossess at $t = 2$ if the contracted

[5] This assumption is necessary for liquidity to play any role and can be justified if endowments are unobservable or if entrepreneurs are only born at $t = 1$. For a detailed discussion of the role of missing insurance markets see Allen and Gale (2004) and Holmstrom and Tirole (2011).

[6] The assumption is shared with Kiyotaki and Moore (1997) and much of the literature building on it.

repayment, r_i, is in default. Following repossession the entrepreneur can still operate the remaining part of his project and collect a proportional income flow at $t = 3$, which will be $2y(1 - \beta_i)$, in expectation. Notice that although all projects may suffer from a temporary income shortage, only capital-poor entrepreneurs require external funding and may thus risk repossession.

Our next assumption is similar to a cash-in-advance constraint: there is no settlement in kind, only in terms of the liquid consumption good. Particularly, debt cannot be settled by the transfer of investment goods. Rather, repossessed collateral needs to be auctioned off, and then the proceeds are used in order to satisfy the lenders.[7] Hence, the $t = 2$ market for investment goods is a fire-sale market. This assumption plays a critical role in the analysis, as it forces the fire-sale price, q_i, below δ if there is not enough liquidity in the market.

Lastly, we assume that bidders in the fire-sale market need to submit orders and the necessary liquidity to execute them *before* the $t = 2$ production process is completed. As a result, $t = 2$ manufactured consumption goods are excluded from fire-sale auction and only liquidity that was stocked up at $t = 1$ can participate. This assumption does not change the qualitative features of our results and is made for realism. In practice we do not observe that consumption drops significantly in order to increase liquidity supply. This is presumably because of a combination of a concave utility function and sufficiently strong time discounting - both of which we do not want to introduce into the model for reasons of tractability.

3. THE CONTRACT

The analysis of this section, and the next, makes no reference to the specifics of country A or B. For brevity, we omit the i index, from the exposition.

Our basic assumption in relation to the contract is that income flows are private information to the borrower.[8] Hence, a capital-poor entrepreneur cannot raise external finance by pledging income flows, and has to pledge his investment good, instead. It is also implied that the lender cannot distinguish a distressed from a non-distressed entrepreneur, so repossession of the collateral can only be conditioned on the observable event of default. Since investment goods lose all their value by $t = 3$, contracts need to be settled by $t = 2$, when the threat of repossession is still effective. The attractive property of this set of assumptions is that it yields a solution that is quite similar to a standard, secured debt contract.

[7]This assumption is shared, among many others, with Allen and Gale (1998), Lorenzoni (2008), Diamond and Rajan (2011) Acharya, Shin and Yorulmazer (2011).

[8]Our setting is a complete contract version of Hart and Moore (1998) with no scope for contract renegotiation. Bolton and Scharfstein (1990) use a closely related set-up to ours where instead of seizing collateral at the interim date, a capital provider can refuse to provide continuation finance.

To see how that works consider, first, a non-distressed borrower with a y income at $t = 2$. The threat of liquidation is effective if the entrepreneur would rather pay r than lose the expected date-3 income associated with the collateral, βy. Hence, our incentive-compatibility constraint is

$$r \leq \beta y, \tag{IC}$$

where β also needs to satisfy a feasibility constraint

$$\beta \in [0, 1]. \tag{FC}$$

Subject to (IC), non-distressed borrowers repay their debt, leaving repossession off the equilibrium path. As for a distressed borrower, income shortage forces him to default, in spite of the heavy losses involved. Lastly, the lenders' participation constraint needs to be satisfied:

$$\pi r + (1 - \pi) \beta q = \rho_1 (1 - \underline{w}). \tag{PC}$$

We assume that (PC) holds with equality due to competition between lenders.

It follows that the optimal contract is an (r, β) pair that maximizes the borrower's consumption (at $t = 3$) subject to the constraints above:

$$\max_{r,\beta} \pi (2y - r) + (1 - \pi) (1 - \beta) 2y, \qquad s.t.\ (IC),\ (PC),\ (FC)\ . \tag{3}$$

(We deal with the entrepreneur's participation constraint below.)

Substituting the binding (PC) into the objective function and re-arranging, we express the final consumption of the capital-poor entrepreneur in terms of β alone:

$$c|_{\underline{w}} = 2y - \rho_1 (1 - \underline{w}) - (1 - \pi) \beta (2y - q). \tag{4}$$

It equals gross income $2y$, net of the cost of external funding, minus the dead-weight loss of external funding, which is the probability of distress, $(1 - \pi)$, times the fraction of the investment pledged as collateral, β, times the per-unit loss from fire sale $(2y - q)$. Since $c|_{\underline{w}}$ is decreasing in β, the solution to the contract problem is the minimal β within the feasible set of the program (3), namely:

Lemma 1 *Let*

$$b = \frac{\rho_1 (1 - \underline{w})}{q (1 - \pi) + \pi y}. \tag{5}$$

If $b \leq 1$, the optimal contract is $\beta = b$; if $b > 1$, a capital-poor entrepreneur cannot obtain external funding.

Proof see Appendix.

By substituting the arbitrage condition (2) into equation (5) we derive b as a function of q alone: $b(q)$. Notice that b is a decreasing function, which plays an important role in the analysis of contagion below, for it means that the supply of fire-sales, $\theta_i (1-\pi) b(q)$ is higher when fire-sale prices are lower. Moreover, $qb(q)$ is also a decreasing function, so that a lower fire-sale price increases the total value of fire-sales and the amount of liquidity that is needed to absorb them.

Let $\underline{q}$ be the positive root of $b(q) = 1$, where (FC) is binding:

$$\underline{q} = \frac{-\pi y + \sqrt{(\pi y)^2 + 4(1-\pi)\delta(1-\underline{w})}}{2(1-\pi)}. \tag{6}$$

Clearly, prices cannot get lower than that for, then, by Lemma (1), no capital-poor entrepreneur can get funding. The only way to avoid this outcome is to set the equilibrium price at $\underline{q}$ and credit-ration some borrowers. Let μ be the probability of obtaining credit. Clearly, if $q > \underline{q}$, $\mu = 1$; if $q = \underline{q}$, $\mu \in [0,1]$, to be determined endogenously in equilibrium. Notice that our assumptions do not prevent credit-rationed entrepreneurs from lending their capital to others.

Lastly, we turn to the entrepreneur's participation constraint. Suppose that credit is rationed and fire-sale prices are down to $\underline{q}$. According to equation (6), when $\underline{w} = 1$, $\underline{q}$ drops to zero and the risk-free lending rate, $\frac{\delta}{\underline{q}}$, tends to infinity. At such a high interest rate entrepreneurs would prefer to lend rather than invest in their own project. Surely, that cannot be an equilibrium. Rather, as entrepreneurs draw capital out of projects, $\underline{q}$ increases via a lowering of $\underline{w}$ in equation (6), until equilibrium is restored. In other words, at high levels of $\underline{w}$ project capitalization is endogenized. To avoid this problem we show:

Lemma 2 *For any combination of the technological parameters π, y and δ, there exists a threshold $\underline{\underline{w}} \in (0,1)$ such that for any $\underline{w} < \underline{\underline{w}}$, $c|_{\underline{w}} > \rho_1 \underline{w}$.*

Proof see Appendix.

We can now decompose

$$c|_{\underline{w}} = [2y - \rho_1 - (1-\pi)\beta(2y - q)] + \rho_1 \underline{w}.$$

Namely, final consumption of the capital-poor entrepreneur is made of endowment (including interest) plus the private value of a project to a capital-poor entrepreneur (to be distinguished from the social value of a project, to be introduced below). It follows from Lemma 2 that given $\underline{w} < \underline{\underline{w}}$ the latter is positive. In fact, we use the result the other way round: for any combination of the technological parameters π, y and δ, we still have one degree of freedom to assume a positive private valuation and select $\underline{w} < \underline{\underline{w}}$ accordingly. Hence:

Assumption 1

$$(2y - \rho_1) - (1-\pi)(2y - \underline{q}) > 0. \tag{A1}$$

The assumption is not restrictive: in it's absence, project capitalization would be endogenized and A1 would hold with equality for high $\underline{w}$'s, introducing two equilibrium "regimes", which is an analytical inconvenience. Notice that $(2y - \rho_1)$ is the private value of a project to a capital-rich entrepreneur, which is greater than the value of a project to the capital-poor entrepreneur and, hence, satisfies his participation constraint as well.

4. A SINGLE COUNTRY BENCHMARK

Suppose there is no cross-country interaction: whoever supplies liquidity, government or speculators, deploys it to that country at $t = 0$ and is not allowed to redeploy it later on. An active government operates like a welfare-oriented speculator: it borrows at $t = 0$ and injects the liquidity ex-post at $t = 1, 2$, into the markets for lending and fire sales. (Only that policy instrument is allowed.) Hence, the ex-post market functions in the same way under competitive and government supply of liquidity. We analyze that market first, and then compare the competitive outcome to that under a welfare-oriented liquidity provider. With a slight abuse of notation we denote by F the aggregate amount of liquidity, regardless of who provides it.

4.1. The ex-post market for liquidity

For any pre-determined F, the ex-post market for liquidity is an aggregation of two spot markets: the $t = 1$ funding market for capital-poor entrepreneurs and the $t = 2$ fire-sale market, both of which are linked via the arbitrage condition (2) and thus cleared through a single price, q. The combined market-clearing condition for a certain realization of θ is:

$$F + (w^n - 1)(1 - \theta) + \theta(1 - \mu)\underline{w} - \theta\mu(1 - \underline{w}) - \theta\mu q(1 - \pi)b(q) \geq 0. \tag{7}$$

On the supply side we have the speculators, the capital-rich entrepreneurs who supply $(w^n - 1)$ liquidity each, and a fraction $(1 - \mu)$ of the capital-poor entrepreneurs who happen to be credit-rationed and are thus willing to lend their endowment at the market rate. On the demand side, there are capital-poor entrepreneurs who are not credit rationed. The last term reflects the fire-sale market: there are θ capital-poor entrepreneurs, of which a fraction μ actually get funding. Of those, a fraction $(1 - \pi)$ will be distressed, so a fraction $b(q)$ of their investment good is repossessed and auctioned off. The amount of liquidity required to absorb the sale is q times $\theta\mu(1 - \pi)b(q)$. In case there is more liquidity available than demand, the clearing condition holds with inequality. Using (1) we can rewrite the clearing condition (7) as

$$F + w_a - (1 - \theta) - \theta\mu[1 + q(1 - \pi)b(q)] \geq 0. \tag{8}$$

PROPOSITION 1 *There exists an ex-post equilibrium in the market for liquidity, with three possible regimes:*

- *If* $\theta < \frac{F}{\underline{q}(1-\pi)}$*, there is a unique equilibrium with excess supply of liquidity:* $q = \delta, \mu = 1$.
- *If* $\frac{F}{\underline{q}(1-\pi)} \leq \theta \leq \frac{F}{\delta(1-\pi)b(\delta)}$*, there are multiple equilibria as follows: (i)* $q = \delta$ *and* $\mu = 1$*, (ii)* $q \in (\underline{q}, \delta)$ *and* $\mu = 1$*, (iii)* $q = \underline{q}, \mu < 1$.
- *If* $\theta > \frac{F}{\delta(1-\pi)b(\delta)}$*, there is a unique equilibrium with credit-rationing:* $q = \underline{q}, \mu < 1$.

In a credit-rationing equilibrium (either the second or the third regime) the amount of credit rationing is:

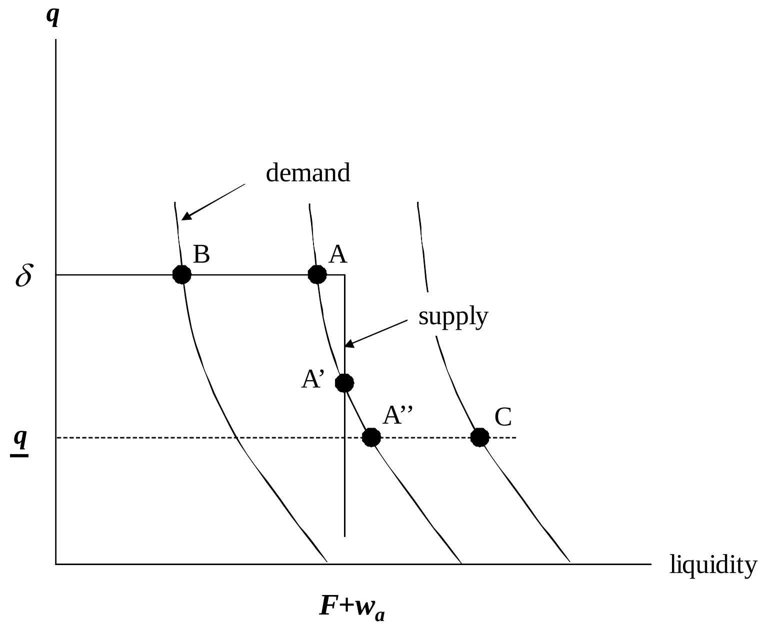
$$\mu = \frac{F + \theta}{\theta \left[1 + \underline{q}\,(1-\pi)\right]}. \tag{9}$$

Proof see Appendix.

Figure 2 provides a diagrammatic exposition of the equilibrium and the existence argument in Proposition (1). The supply of liquidity is perfectly elastic at the "fair" price δ up to the point $F + w_a$ when liquidity supply is exhausted. As noted, the demand for liquidity is decreasing in the fire-sale price, q. Three possible realizations of θ are plotted: for the low and high realizations there is a unique equilibrium at points B and C, respectively, while for the interim case there are multiple equilibria at points A, A' and A''. (Notice that points C and A'' represent the equilibrium price but not the quantity, due to credit rationing.)

Figure 2

The ex-post market for liquidity



Since asset repossession destroys value, the equilibrium points in the interim case are Pareto ranked: point A dominates point A' which dominates point A''. Hence, the government should try to coordinate expectations towards point A:

PROPOSITION 2 *In case of multiple equilibria, a policy that guarantees a fire-sale price of δ eliminates the Pareto-dominated equilibrium points at a zero fiscal cost.*

Proof. Let q^e be the expected fire-sale price. Then, equilibrium is determined by substituting

$$b_{GR} = \frac{\frac{\delta}{q^e}\left(1 - \underline{w}\right)}{q^e\left(1 - \lambda\pi\right) + \lambda\pi y}$$

into the equilibrium condition (8). Clearly, the demand for liquidity is no longer sensitive to the actual market price, q, and equilibrium is unique. Since $q^e = \delta$ is an equilibrium, it must be the only one. To the extent that the government guarantees a price of δ, the guarantee-holders will not exercise them and the fiscal cost of the policy is zero. Notice that this argument does not hold for θs where $\underline{q}$ is the unique equilibrium price. ■

ASSUMPTION 2 *In case of multiple equilibria, the government coordinates expectations towards the Pareto-dominating point.*

It follows that there exist a unique critical point,

$$\theta^* = \frac{F}{\delta\left(1 - \pi\right) b\left(\delta\right)} \tag{10}$$

such that the economy is in "financial crisis" whenever $\theta > \theta^*$. Crisis is characterized by a sharp increase in repossessions, a drop in the fire-sale price and by a "credit crunch": rationing, tighter collateral requirements and a higher cost of borrowing. Crisis is also characterized by contagion, which has the most dramatic effect around the critical state, θ^*. Just a tiny increase in the incidence of capital-poor entrepreneurs around θ^* dramatically worsens the extent of repossessions for all other capital-poor entrepreneurs. Contagion is intimately related to the negative slope of the demand for liquidity. For once fire-sale prices start to drop, borrowers have to pledge a larger fraction of their project as collateral, which generates even more fire sales. Another implication of the negative slope of the demand for liquidity is a "multiplier effect": the ratio between the change in equilibrium magnitudes and the initial "shocks" that have generated them is unbounded. Obviously, this characteristic is an immediate implication of a market where both supply and demand slope in the same direction.

4.2. Competitive supply of liquidity

Suppose that only speculators provide liquidity. At $t = 0$, when they make their decisions, they must realize that they would make losses in case no financial crisis develops at $t = 1$, namely when $\theta < \theta^*$. To break even, the return on liquidity in crisis needs to exceed the ex-ante cost of liquidity:

ASSUMPTION 3

$$\rho_0 < \frac{\delta}{\underline{q}}. \tag{A3}$$

It follows that the equilibrium probability of crisis, $\left(1-\frac{\theta^*}{\overline{\theta}}\right)$, is determined by the break-even condition:

$$\rho_0 = \frac{\theta^*}{\overline{\theta}} + \left(1-\frac{\theta^*}{\overline{\theta}}\right)\frac{\delta}{\underline{q}}. \tag{11}$$

Given the critical θ^*, F is determined via equation (10), with the following important implication:[9]

Lemma 3 *The competitive-equilibrium probability of financial crisis, R, is strictly positive at*

$$R = \frac{\rho_0 - 1}{\frac{\delta}{\underline{q}} - 1}.$$

Proof. Follows immediately from equation (11). ■

4.3. Welfare-oriented supply of liquidity

Liquidity is a public good. To substantiate this claim, we account social welfare across various levels of liquidity, and compare the welfare-maximizing F to the competitive one. Since competitive liquidity is characterized by a break-even condition (11), the government does not break even when it deviates from competitive liquidity: it profits (in expectations) when it "monopolizes" the liquidity market and loses when it provides liquidity in excess of the competitive amount. Trading losses (profits) are funded by taxes (transfers) which are non-distortionary up to the point T. Assume, for the time being, that the constraint on T is not binding, i.e., the government could provide enough liquidity $\overline{F} \equiv \overline{\theta}\delta\left(1-\pi\right)b\left(\delta\right)$ so as to eliminate any crisis:

$$\overline{F} \leq T \tag{12}$$

To derive the social-welfare function, $SW\left(\theta^*\right)$, we take expectations over the entrepreneurs' final consumption, c, conditional on liquidity, which is conveniently expressed in terms of the critical θ^*, using equation (10). Doing so we net out all transfers across capital-poor and capital-rich entrepreneurs, as well as the the government's trading profits and taxes. As a result, we account repossessions at their social cost, δ, rather than their private cost $\underline{q}$.

Denote the social value of a project in and out of financial crisis by $v\left(\underline{q}\right)$ and $v\left(\delta\right)$, respectively, where

$$v\left(\underline{q}\right) \equiv (2y-1) - (1-\pi)(2y-\delta), \tag{13}$$

$$v\left(\delta\right) \equiv (2y-1) - (1-\pi)\,b\left(\delta\right)(2y-\delta), \tag{14}$$

$$\Delta \equiv v\left(\delta\right) - v\left(\underline{q}\right). \tag{15}$$

[9] Notice that the probability of crisis is independent of distributional assumptions.

These are used in order to construct the social-welfare function and its derivative (see Appendix for more detail):

$$SW'(\theta^*) = [\mu^*\Delta + (1-\mu^*)\, v(\delta)]\, \frac{\theta^*}{\overline{\theta}} + \left(1 - \frac{\theta^*}{\overline{\theta}}\right) \mu_F v\left(\underline{q}\right) - (\rho_0 - 1)\, F_{\theta^*}. \quad (16)$$

where

$$\mu^* \equiv \frac{1 + \delta(1-\pi)\, b(\delta)}{1 + \underline{q}(1-\pi)} = \mu_F + \mu_\theta. \quad (17)$$

$$\mu_F \equiv \frac{d(\theta\mu)}{d\theta^*} = \frac{\delta(1-\pi)\, b(\delta)}{1 + \underline{q}(1-\pi)}, \quad (18)$$

$$\mu_\theta \equiv \frac{d(\theta\mu)}{d\theta} = \frac{1}{1 + \underline{q}(1-\pi)}, \quad (19)$$

$$F_{\theta^*} \equiv \frac{dF}{d\theta^*} = \delta(1-\pi)\, b(\delta). \quad (20)$$

The derivative (16) has quite an intuitive interpretation. The last term nets out the marginal cost of liquidity: to move the critical state by $d\theta^*$ takes F_{θ^*} liquidity that needs to be borrowed at market rate $(\rho_0 - 1)$. As a result, θ^* capital-poor entrepreneurs, measured by the marginal density, $1/\overline{\theta}$, are lifted out of crisis, which will allow them to decrease the amount of pledged collateral from 1 to $b(\delta)$ and increase the social valuation of their projects by Δ. That effect, however, is relevant only to the μ^* fraction of capital-poor entrepreneurs that were not credit rationed when the θ^* state was in crisis. For the $1-\mu^*$ fraction of capital-poor entrepreneurs who were credit rationed the increase in social valuation would be the entire social value of their project $v(\delta)$. But there is an additional effect on those states of nature that remain in crisis, with a probability of $\left(1 - \frac{\theta^*}{\overline{\theta}}\right)$. Due to the greater supply of liquidity, the incidence of credit rationing drops by μ_F, which allows the respective entrepreneurs to increase their social contribution by $v\left(\underline{q}\right)$ (which is less than $v(\delta)$ since collateral requirements are still 1 during a crisis).

The analysis is somewhat complicated by the fact that SW can be either convex or concave.[10] From (16) we can directly compute

$$SW''(\theta^*) = \frac{1}{\overline{\theta}} \left[\Delta + (1 - \mu^* - \mu_F)\, v\left(\underline{q}\right)\right]. \quad (21)$$

Nevertheless, in both cases (as well as in the two-country analysis below), solutions reveal a strong tendency towards a corner solution.

Proposition 3 *For a sufficiently high T such that (12) holds, the socially optimal level of liquidity is at the corner $\theta^* = \overline{\theta}$ resulting in a zero probability of financial crisis.*

Proof see Appendix.

[10] For example, for the technological parameters $y = 1.25$, $\delta = 0.5$ and $\pi = 0.75$ we get $SW'' > 0$ for $\underline{w} = 0.2$ and $SW'' < 0$ for $\underline{w} = 0.1$.

It follows that in a competitive equilibrium there is under-provision of liquidity and, as a result, an excessively high probability of financial crisis. The main driver behind this result is the stark difference between the break-even condition (11) and the first-order condition (16). While the former is determined by the market value of the discontinued investment goods, δ and $\underline{q}$, the latter is determined by their (social) continuation value, $v(\delta)$ and $v(\underline{q})$. Clearly, several missing markets distort prices away from social valuations. Could the entrepreneur buy back the repossession rights from the creditor he would, but a shortage of liquidity prevents him from doing so. Could he buy insurance that provides him with liquidity in distress and facilitate the buy-back, he would, but that market is missing as well. Indeed, could he buy insurance against being capital poor (and eliminate the need to take secured credit) he would, but that market is also missing.

Another implication of the gross inefficiency of the competitive equilibrium is the strong tendency of a welfare-enhancing policy to be pushed towards a corner solution. Remember that there is no economic distress in our model, and that (up to T) a policy can mitigate financial distress by supporting fire-sale prices – without creating any tax distortions. But then, if the policy generates welfare while applied on a narrow margin (small θ^*), it would add even more value when applied on a wide margin, which generates, in many cases, a convex welfare function and, always, a corner solution.

4.4. A mix of private and public liquidity

What happens if both speculators and government supply liquidity? Let θ^* be the total amount of liquidity provided jointly. There can be two equilibrium regimes:

$$\begin{aligned} \frac{\theta^*}{\overline{\theta}} &> (1-R), \text{ with no active speculators,} \\ \frac{\theta^*}{\overline{\theta}} &= (1-R), \text{ with active speculators.} \end{aligned}$$

In the first equilibrium regime the probability of crisis is too small for speculators to satisfy the break-even condition (11), so they do not participate. In the second, if the government provides less liquidity than implied by θ^*, the speculators top up the government's liquidity up to the point where the break-even condition (11) holds. It follows that equilibrium liquidity is neutral in the government's liquidity policy, or, to put it differently:

LEMMA 4 *For equilibria where the speculators actively supply liquidity, an increase in public liquidity crowds out private liquidity one-for-one.*

Proof. Follows directly from Lemma 3. ■

At some point, however, the speculators withdraw from the market completely. That happens when public liquidity increases θ^* to the point where $\frac{\theta^*}{\overline{\theta}} > (1-R)$. From that point on, any increase in public liquidity would increase total liquidity by an equal amount, with a corresponding decrease in the probability of crisis. The practical implication of Lemma 4 is that the government needs to "nationalize" the market for liquidity, i.e., become the sole liquidity supplier, before it can have any welfare effect.

5. COMPETITIVE EQUILIBRIUM WITH TWO COUNTRIES

We now restore the country index $i = A, B$. We denote by F_i domestic liquidity, which is deployed to country i at $t = 0$ and cannot be redeployed thereafter. As we shall see below, F_i should be interpreted as publicly provided or at least publicly subsidized liquidity. F is non-territorial liquidity or hot money, deployed to crisis economies only upon the realization of (θ_A, θ_B) at $t = 1$. We follow the same structure of exposition as before, starting with the ex-post market where both F_i and F are pre-determined.

5.1. The ex post market for liquidity

There are four equilibrium regimes: no crisis in either country (NC), a regional crisis in one country only, either country A $(RC - A)$ or country B $(RC - B)$ and, finally, a "systemic" crisis in both countries (SC). We maintain all previous assumptions, particularly Assumption 2, so that in case of multiple equilibria expectations are coordinated towards the Pareto-dominant one. Like before, $q_i = \delta$ out of crisis and $q_i = \underline{q}$ in crisis. Credit is rationed in the latter case. We characterize the four regimes with the aid of Figure 3 that partitions the space set $(\theta_A, \theta_B) \in [0, \overline{\theta}] \times [0, \overline{\theta}]$ accordingly.

There will be a NC equilibrium *iff* for *both* $i = A, B$ *both* conditions hold:

$$\Sigma_i \theta_i (1 - \pi) \delta b(\delta) \leq F + \Sigma_i F_i, \tag{22}$$

$$\theta_i (1 - \pi) \delta b(\delta) \leq F + F_i. \tag{23}$$

Namely, there is enough liquidity in the world (domestic plus hot money) to satisfy the entire demand by both countries at $q = \delta$, subject to the additional constraint that each country's demand can be satisfied without the domestic liquidity of its neighbor. Like before, it is convenient to express the equilibrium conditions in terms of critical θs. For that purpose we define:

$$\theta_i^* \equiv \frac{F_i}{(1 - \pi) \delta b(\delta)},$$

$$\theta^* \equiv \frac{F}{(1 - \pi) \delta b(\delta)},$$

$$\widehat{\theta} = \Sigma_i \theta_i^* + \theta^*,$$

and reformulate conditions (22) and (23) :

$$\Sigma_i \theta_i \leq \widehat{\theta}, \qquad \text{(NC)}$$

$$\theta_i \leq \theta_i^* + \theta^*.$$

There will be a $RC - A$ equilibrium *iff*

$$\begin{aligned} \theta_A &> \theta_A^* + \theta^*, \\ \theta_B &\leq \theta_B^*. \end{aligned} \tag{RC-A}$$

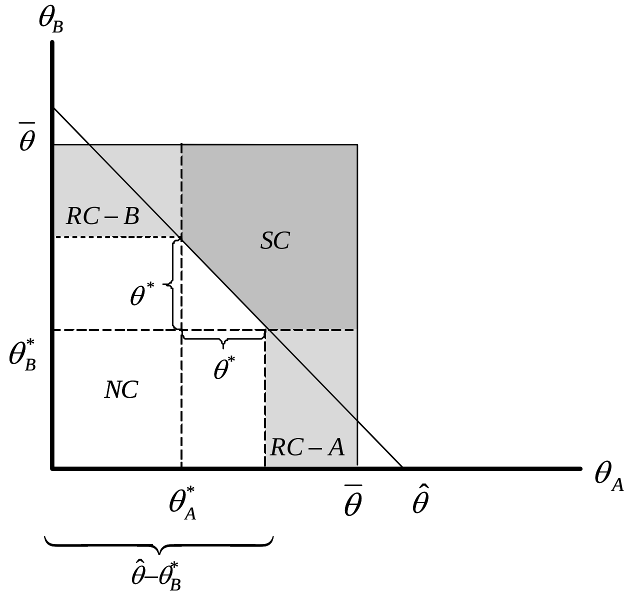
Namely, country A cannot satisfy its demand (at $q = \delta$) using its own liquidity plus the entire stock of hot money, while country B can satisfy its demand relying on its domestic liquidity alone. In this case, country-A's fraction of poor entrepreneurs that are allocated with funding can be calculated from a condition similar to (8) in the one-country case, taking into account that the pool of available liquidity is made of domestic liquidity, F_A plus hot money, F. Hence,

$$\mu_A = \frac{F_A + F + \theta_A}{\theta_A \left[1 + (1 - \pi)\, \underline{q}\right]}. \tag{24}$$

The $RC - B$ area is symmetrically defined.

Figure 3

The four equilibrium regimes



There will be a SC equilibrium *iff*

$$\begin{aligned} \Sigma_i \theta_i &> \widehat{\theta}, \\ \theta_i &> \theta_i^*. \end{aligned} \tag{SC}$$

Namely, there is not enough liquidity in the world to satisfy both countries' demand (at $q = \delta$) and, additionally, neither country can satisfy its demand with its domestic liquidity alone. Hence $q_i = \underline{q}$ and both countries suffer from credit rationing

$$\mu_i = \frac{F_i + \theta_i + \eta_i F}{\theta_i \left[1 + (1 - \pi)\, \underline{q}\right]},$$

where $\eta \equiv \eta_A$ is the share of hot money allocated to country A, and $\eta_B = 1 - \eta$ is allocated to country B.

So far, our assumptions impose no structure on η apart from the obvious $0 \leq \eta \leq 1$. We therefore assume a simple linear allocation rule

Assumption 4 *A countries' share in hot money is proportional to the excess of domestic demand to domestic supply of liquidity:*

$$\frac{\eta}{1-\eta} = \frac{\theta_A - \theta_A^*}{\theta_B - \theta_B^*}. \tag{A4}$$

Solving for η we get:

$$\eta = \frac{\theta_A - \theta_A^*}{(\theta_A - \theta_A^*) + (\theta_B - \theta_B^*)}. \tag{25}$$

Notably, a country's allocation of hot money is increasing in its own excess demand and decreasing in its neighbor's excess demand. In the analysis below we shall give special attention to the allocation of hot money on the margins of the SC area.

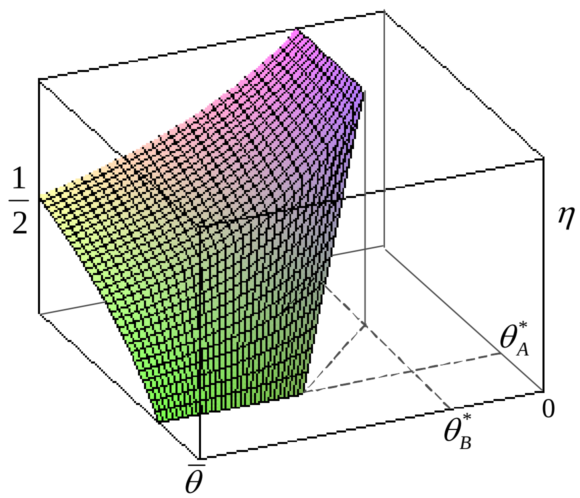
$$\eta = \begin{cases} 0 & \text{when} \quad \theta_A = \theta_A^* & \text{(on the } RC - B \text{ edge)} \\ \frac{\theta_A - \theta_A^*}{\widehat{\theta} - \left(\theta_A^* + \theta_B^*\right)} & \text{when} \quad \theta_B = \widehat{\theta} - \theta_A & \text{(on the } NC \text{ edge)} \\ 1 & \text{when} \quad \theta_B = \theta_B^* & \text{(on the } RC - A \text{ edge)} \end{cases}.$$

Figure 4 plots the η function over the SC region for the case of symmetric supply of domestic liquidity, namely $\theta_A^* = \theta_B^*$.

Figure 4

The η function with symmetric domestic liquidity

$\overline{\theta} = 0.6, \widehat{\theta} = 0.5, \theta_A^* = \theta_B^* = 0.15$



Using the characterization (NC), (RC-A) and (SC) of the regions for the various equilibrium regimes, we can calculate their respective probabilities:

$$\pi^{NC} = \frac{1}{\overline{\theta}^2}\left[\left(\widehat{\theta}-\theta_B^*\right)\left(\widehat{\theta}-\theta_A^*\right)-\frac{1}{2}\left(\widehat{\theta}-\theta_A^*-\theta_B^*\right)^2\right], \tag{26}$$

$$\pi^{RC-A} = \frac{1}{\overline{\theta}^2}\left(\overline{\theta}-\widehat{\theta}+\theta_B^*\right)\theta_B^*, \tag{27}$$

$$\pi^{RC-B} = \frac{1}{\overline{\theta}^2}\left(\overline{\theta}-\widehat{\theta}+\theta_A^*\right)\theta_A^*, \tag{28}$$

$$\pi^{SC} = \frac{1}{\overline{\theta}^2}\left[\left(\overline{\theta}-\theta_A^*\right)\left(\overline{\theta}-\theta_B^*\right)-\frac{1}{2}\left(\widehat{\theta}-\theta_A^*-\theta_B^*\right)^2\right]. \tag{29}$$

Since the object of our analysis is the welfare effect of hot money we restrict attention to equilibria where speculators actively supply hot money so that $\theta^* > 0$. As we shall see below, our analysis is meaningful only for equilibria where changes in hot money affect the likelihood of both systemic and regional crisis. To that end, we also restrict attention to equilibria where regional crisis occurs with a positive probability. Hence, for country A, $\overline{\theta} - (\theta_A^* + \theta^*) > 0$. It is convenient to express the above two conditions in terms of total liquidity, $\widehat{\theta}$ rather than hot money, θ^*. Hence, we restrict attention to the "relevant area" as defined below:

Definition 1 *The "relevant area" is the set of all $\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right)$ combinations such that*

$$\widehat{\theta} - (\theta_A^* + \theta_B^*) > 0, \tag{D.1}$$

$$\overline{\theta} - \left(\widehat{\theta} - \theta_i^*\right) > 0. \tag{D.2}$$

5.2. Competitive supply of liquidity

Since hot money can be deployed in both countries, speculators profit when at least one country is in crisis. It follows that, given θ_A^* and θ_B^*, speculators select the amount of hot money such that

$$\pi^C = R, \tag{30}$$

where

$$\pi^C \equiv \pi^{SC} + \pi^{RC-A} + \pi^{RC-B}. \tag{31}$$

The two-country case differs materially from the one-country case in that domestic and international liquidity are not perfect substitutes: just notice that the rate of return on hot money is $\pi^C\left(\frac{\delta}{q}-1\right)$ while the rate of return on domestic, say country A, liquidity is only $\left(\pi^{SC}+\pi^{RC-A}\right)\left(\frac{\delta}{q}-1\right)$. Clearly, domestic liquidity cannot be competitively supplied. Hence, the most straight-forward interpretation of domestic liquidity is government-supplied liquidity, with trading losses funded by taxes. Alternatively, one may think of private supply of domestic liquidity, say by big domestic financial institutions, where the low rate of return is compensated by some commercial advantages, say, monopoly rights in the supply of certain financial services. These should be considered as an effective tax on the domestic population. Or, one can interpret domestic liquidity as being supplied by domestic speculators who are barred from speculating abroad, in which case the tax falls on these speculators rather than the general population.

The key economic implication of imperfect substitutability is that domestic liquidity crowds out international liquidity, but only partially so. To demonstrate the result, substitute (27)-(29) into (31):

$$\pi^C = 1 - \frac{1}{2}\left[\left(\frac{\widehat{\theta}}{\overline{\theta}}\right)^2 - \left(\frac{\theta_A^*}{\overline{\theta}}\right)^2 - \left(\frac{\theta_B^*}{\overline{\theta}}\right)^2\right], \tag{32}$$

and then (30) into (32). Solving for $\widehat{\theta}$

$$g\left(\theta_A^*, \theta_B^*\right) = \sqrt{\theta_A^{*2} + \theta_B^{*2} + 2\overline{\theta}^2\left(1 - R\right)}, \tag{33}$$

we derive $g\left(\theta_A^*, \theta_B^*\right)$, the competitive-equilibrium amount of total liquidity.

It follows that

$$\frac{\partial g\left(\theta_A^*, \theta_B^*\right)}{\partial \theta_i^*} = \frac{\theta_i^*}{g\left(\theta_A^*, \theta_B^*\right)}, \quad \text{for} \quad i = A, B, \tag{34}$$

which allows for the characterization of partial crowding out.

Lemma 5 *Suppose hot money is determined in a competitive equilibrium, so that $\widehat{\theta}$ is given by (33). Then a unilateral increase in, say, country A's domestic liquidity, θ_A^* (holding country-B's domestic liquidity, θ_B^*, constant), i) crowds out international liquidity, θ^* but ii) only partially (i.e. by less than the increase in θ_A^*), so that iii) the pool of liquidity available to country A, $\theta_A^* + \theta^* = \widehat{\theta} - \theta_B^*$, increases, while iv) the pool of liquidity available to country B (i.e. $\theta_B^* + \theta^* = \widehat{\theta} - \theta_A^*$) decreases. v) If at a symmetric point, $\theta_A^* = \theta_B^*$, both countries increase domestic liquidity by the same amount, $\widehat{\theta}$ increases.*

Proof. Using assumption (A5.1) in equation (34) it follows that

$$0 \leq \frac{\partial g\left(\theta_A^*, \theta_B^*\right)}{\partial \theta_i^*} \leq \frac{1}{2} \quad \text{for} \quad i = A, B,$$

from which points *i)-iv)* follow. Point *v)* follows from

$$0 \leq \left.\frac{\partial g\left(\theta_A^*, \theta_B^*\right)}{\partial \theta_A^*}\right|_{\theta_A^* = \theta_B^*} + \left.\frac{\partial g\left(\theta_A^*, \theta_B^*\right)}{\partial \theta_B^*}\right|_{\theta_A^* = \theta_B^*} \leq 1. \tag{35}$$

■

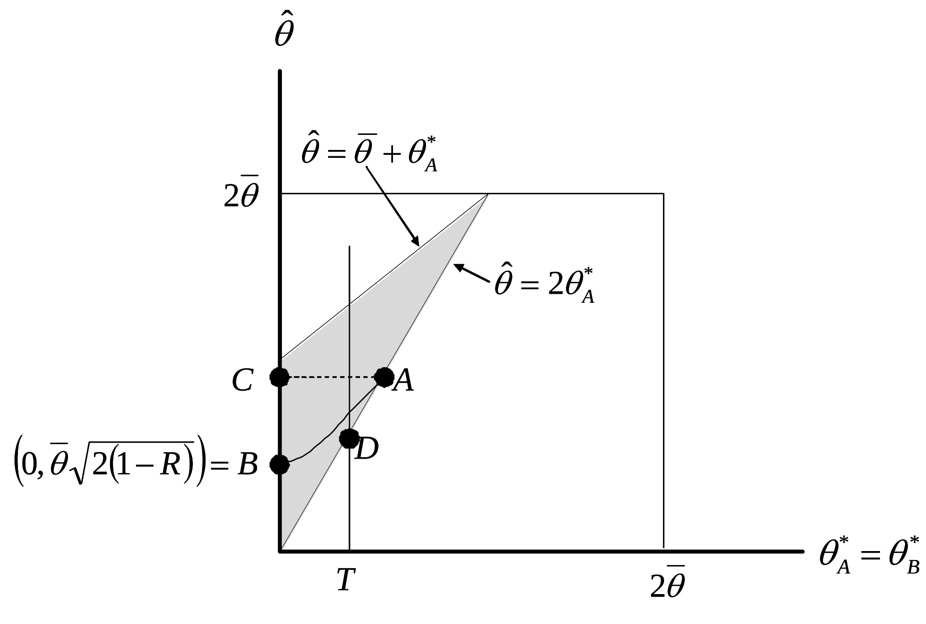
To gain some insight into Lemma 5 suppose, by way of contradiction, that there is full crowding out: in Figure 3 this would correspond to increasing country-A's liquidity, θ_A^*

(leaving θ_B^* constant), and decreasing hot money, θ^*, by the same amount, so that total liquidity, $\widehat{\theta}$, remains the same. Clearly, the $RC-B$ area, which represents the probability of country-B regional crisis, expands as the pool of liquidity available to it, $\theta_B^*+\theta^*$, falls. It follows that the probability of crisis anywhere in the world has actually increased, which provides speculators with an incentive to increase their supply of liquidity. It follows that in equilibrium (after the adjustment) hot money, θ^*, falls by less than the increase in θ_A^*. Hence, the pool of liquidity available to country A, $\theta_A^*+\theta^* = \widehat{\theta}-\ \theta_B^*$, would increase while the pool of liquidity available to country B, $\theta_B^* + \theta^* = \widehat{\theta}-\ \theta_A^*$, decreases.

To summarize the results we provide, in Figure 5, a diagrammatic illustration of the competitive, symmetric, equilibrium. The shaded area is where conditions (D.1) and (D.2) in Definition 1 hold. The $B-A$ curve plots the equilibrium amount of total liquidity, $\widehat{\theta}$, as a function of domestic (symmetric) liquidity, namely the $g(\theta_A^*, \theta_B^*)$ function as defined in (33). Notice that at zero domestic liquidity, total liquidity is $\widehat{\theta} = \overline{\theta}\sqrt{2(1-R)}$. Point *v)* of Lemma 5 demonstrates that the slope of the $B-A$ curve is smaller than one, so that for a B point below $\overline{\theta}$ (namely $R \geq \frac{1}{2}$) the $B-A$ curve lies within the shaded area and intersects with its lower boundary. (Points C and D in Figure 5 are for subsequent reference.)

Figure 5

The "relevant area", total liquidity and symmetric-regional liquidity



For reasons of tractability we limit the analysis, from now on, to the symmetric case. We also bring back the constraint on government borrowing, T_i, which until now did not play any role in the analysis.

5.3. The unilateral incentive to fragment

To analyze the incentive of country-A's government to expand domestic liquidity unilaterally, we analyze variations in country-A's social welfare, $SW\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right)$ (see Appendix for technical detail), holding θ_B^* constant, but allowing total liquidity to be determined competitively according to the g function as defined in (33).

PROPOSITION 4 *The unilateral welfare effect of increasing domestic liquidity (36) is always positive. It follows that in an uncoordinated equilibrium, both countries increase domestic liquidity up to the corner solution T.*

Proof see Appendix.

To gain some intuitive insight into the result, we decompose the welfare effect to a "pure" fragmentation effect and a liquidity effect:

$$\frac{dSW\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right)}{d\theta_A^*} = SW_{\theta_A^*} + \frac{\partial g\left(\theta_A^*, \theta_B^*\right)}{\partial \theta_A^*} SW_{\widehat{\theta}}, \tag{36}$$

where SW_{θ_i} and $SW_{\widehat{\theta}}$, are partial derivatives of social welfare with respect to θ_i^* and $\widehat{\theta}$, respectively. The liquidity effect is positive due to the, basic, under-provision of liquidity in our model.

More involved is the fragmentation effect, which isolates the effect of substituting hot money with domestic liquidity, holding total liquidity, $\widehat{\theta}$, constant so that there is one-to-one crowding out of hot money by country-A domestic liquidity. With θ_B^* also constant, that leads to a reduction in the liquidity available to country B, $\theta_B^* + \theta^*$. As a result, realizations with $\theta_B = \theta_B^* + \theta^*$, along the $RC-B$ and NC margin, move from the no-crisis to regional country-B crisis (see Figure 3). Notice that, along this margin, liquidity in country A stands idle while Country B is in crisis. Obviously, this effect is immaterial to country A, but has negative implications for country B; hence the externality, and the prospect for coordination that plays a central role in the welfare analysis below. Though the pool of liquidity available to country A, $\theta_A^* + \theta^*$, stays the same, more of that pool is reserved for country-A only, so realizations with $\theta_A = \theta_A^*$, along the $RC - B$ and SC margin, move from systemic crisis to country-B only crisis, and thus no crisis in country A. This second effect is welfare enhancing for country A, providing an incentive for unilateral fragmentation.

More formally, evaluating $SW_{\theta_A^*}$ at the symmetric point $\theta_A^* = \theta_B^*$, we can write (see Appendix for the technical details):

$$\begin{aligned} SW_{\theta_A^*}\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right) &= \frac{1}{\overline{\theta}}\left[\mu^*\Delta + (1-\mu^*)\, v(\delta)\right]\left[\theta_A^* \frac{\overline{\theta} - \left(\widehat{\theta} - \theta_A^*\right)}{\overline{\theta}}\right] \\ &+ \left[\frac{1}{2}\pi^{SC} + \left(\widehat{\theta} - 2\theta_A^*\right)\overline{\eta}_{\theta_A^*}\right]\mu_F v\left(\underline{q}\right) \\ &+ \left[\frac{1}{2}\pi^{SC} - \left(\widehat{\theta} - 2\theta_A^*\right)\overline{\eta}_{\theta_A^*} + \pi^{RC-A}\right]\left(\frac{\delta}{\underline{q}} - 1\right) F_{\theta^*} \\ &- (\rho_0 - 1)\, F_{\theta^*} + \frac{1}{2} L_{\theta_A^*}\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right), \end{aligned} \tag{37}$$

where $\overline{\eta}$ is the expected value of η over the SC equilibrium regime and $\overline{\eta}_{\theta_A^*}$ denotes its partial derivative with respect to θ_A^* (both derived and stated in the Appendix). The function $L\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right)$ and its partial derivative $L_{\theta_A^*}$ are defined in the Appendix; its economic interpretation is discussed in detail in the next section.

More accurately, on the first line of equation (37), and conditional on the marginal realization θ_A^*, the probability of crisis drops from $\left[\overline{\theta} - \left(\widehat{\theta} - \theta_A^*\right)\right]/\overline{\theta}$ to zero. To get the (positive) welfare effect, multiply by $\left[\mu^*\Delta + (1-\mu^*)\, v(\delta)\right]$, already interpreted in relation to equation (16). The second line of (37) captures the effect of credit rationing. The actual allocation of liquidity to country A is its own, plus its share of hot money: $\theta_A^* + \eta\left(\widehat{\theta} - \theta_A^* - \theta_B^*\right)$. In the $RC-A$ region, $\eta = 1$; with full crowding out (namely the boundary of the $RC-A$ and the SC areas), it is still the case that $\eta = 1$, so the substitution of hot money by domestic liquidity has no effect on credit availability. But within the SC region, $\overline{\eta} = \frac{1}{2}$, so domestic liquidity only half crowds-out hot money, increasing credit availability. On top, we add the (negative) effect that as country A moves away from the symmetric point, it competes less aggressively for its share in hot money; hence $\overline{\eta}_{\theta_A^*}$. To get the welfare effect, multiply by $\mu_F v\left(\underline{q}\right)$, also interpreted in relation to equation (16). The next line captures the effect of lower trading losses (to speculators) as a result of the fall in hot money. Also, by being less aggressive on its share in hot money, country A saves on trading losses, which has a positive welfare effect. The last line captures the increase in the direct cost of liquidity.

6. WELFARE ANALYSIS OF THE TWO-COUNTRY CASE

It is already clear from the discussion above that in the case of a unilateral expansion of liquidity, country A deprives country B from liquidity without internalizing the welfare effect. Obviously, as country B responds in kind it inflicts the same "beggar-thy-neighbor" externality on country A. Clearly, the prospects of coordination should play a central role in our welfare analysis.

Once coordination is considered, the issue of conditioning the allocation of liquidity on the realized shocks (θ_A, θ_B) arises. Rather than setting aside liquidity that can be deployed only domestically, the countries could agree to pool their resources and negotiate, to their mutual benefit, a conditional allocation rule. Section 6.1 deals with this issue and derives the triage. The analysis highlights that this optimal allocation rule requires a degree of commitment that is not readily available among sovereigns. This may explain why the triage is not observed in practice. We therefore investigate, in Section 6.2, whether unconditional coordination can enhance welfare.

In both these sections, and in order to allow for a meaningful analysis of fragmentation out of the competitive equilibrium, one has to allow for non-regional liquidity, $\widehat{\theta} - (\theta_A^* + \theta_B^*)$, which may differ from the competitive level of hot money, $g(\theta_A^*, \theta_B^*)$. Since competitive hot money is defined by the break-even condition (11), it follows that there will be non-zero trading profits, or losses (in expectation), on the non-competitive pooled liquidity, which will have to be funded by the participating countries. To that end, and as part of their coordination (whether conditional or unconditional), the countries would have to set up a joint liquid fund, and deal with the trading losses (or profits) via lump-sum taxes. We denote the expected net payoff on that fund by $L\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right)$. This term already appears in the derivative of the welfare function (37). Since, by construction, $L\left[\theta_A^*, \theta_B^*, g(\theta_A^*, \theta_B^*)\right] = 0$, it played only a technical role there in facilitating the decomposition of the total effect to the liquidity and the fragmentation effects. We conclude with a welfare analysis of unilateral fragmentation.

6.1. Coordinated state-contingent allocation of liquidity: The triage

Suppose that the two governments agree, ex ante, a certain amount of a joint liquid fund of a magnitude $\widehat{\theta}$ (to which they contribute equally) and an allocation rule contingent upon the realization (θ_A, θ_B), so as to maximize their expected welfare.

Proposition 5 *Under the optimal triage rule, both countries should avoid crisis if feasible, i.e., if*

$$\theta_A + \theta_B \leq \widehat{\theta}.$$

There should be a country-B regional crisis, with Country A getting just enough liquidity to avoid crisis

$$\begin{aligned} \text{if } \theta_i &\leq \widehat{\theta}, \text{ and } \theta_A > \theta_B, \\ \text{or if } \theta_A &\leq \widehat{\theta} \text{ and } \theta_B > \widehat{\theta}. \end{aligned}$$

There should be a symmetric country-A regional crisis

$$\begin{aligned} \text{if } \theta_i &\leq \widehat{\theta}, \text{ and } \theta_A < \theta_B \\ \text{or if } \theta_A &> \widehat{\theta} \text{ and } \theta_B \leq \widehat{\theta}. \end{aligned}$$

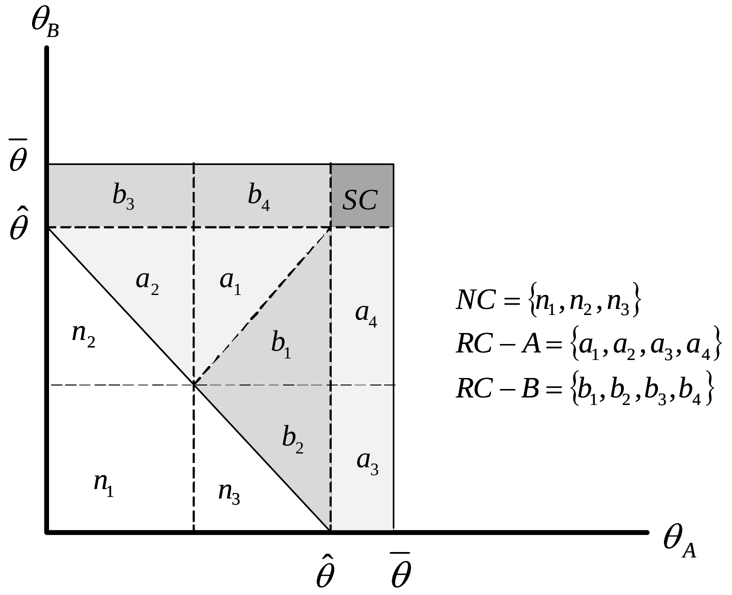
There should be a systemic crisis only when it is not feasible to contain crisis regionally, i.e., when $\theta_A > \widehat{\theta}$ and $\theta_B > \widehat{\theta}$. In this case the allocation of liquidity does not matter.

Proof see Appendix.

Figure 6 provides a diagrammatic exposition of the triage rule. For realizations in the NC area there is enough liquidity to rescue both countries. For realizations in the SC area there is not enough liquidity to rescue either country. For all other realizations there is enough liquidity to rescue only one country. Since the social cost of crisis increases in the number of capital-poor entrepreneurs, rescue should be prioritized, with the more badly-injured (higher θ) country coming first. Hence, in areas b_1 and b_2, where $\theta_A > \theta_B$ country A should be saved, leaving country B "to sink" (but still, directing towards country B all the liquidity that is not used in the rescue of country A so as to avoid credit-rationing as much as possible). In areas a_3 and a_4 although it is still the case that $\theta_A > \theta_B$, country A cannot be saved even if it received the entire available liquidity. Only in that case should country B, which is less badly-injured than country A be rescued, leaving country A to its fate. A symmetric argument applies when $\theta_A < \theta_B$.

Figure 6

Crisis areas for the triage, given $\widehat{\theta}$



To some extent, the welfare implications of an unconditional policy can be analyzed in terms of deviation from the triage. To that end, we compare, in Table 1, the triage with two extreme cases of unconditional arrangements (with the same $\widehat{\theta}$): full fragmentation where all the available liquidity is regionalized, $\theta_A^* = \theta_B^* = \widehat{\theta}/2$, and complete pooling where all liquidity is pooled, $\theta_A^* = \theta_B^* = 0$ and $\theta^* = \widehat{\theta}$ (and allowed to flow freely, ex post).

Table 1

Crisis regimes under the triage and unconditional fragmentation

Area	Triage	Unconditional fragmentation	Unconditional pooling
NC	$\{n_1, n_2, n_3\}$	n_1	$\{n_1, n_2, n_3\}$
$RC-B$	$\{b_1, ...b_4\}$	$\{n_2, a_2, b_3\}$	–
$RC-A$	$\{a_1, ..., a_4\}$	$\{n_3, b_2, a_3\}$	–
SC	SC	$\{a_1, a_4, b_1, b_4, SC\}$	$\{a_1, ..., a_4, b_1, ..., b_4, SC\}$

The main advantage of an unconditional pooling policy is that it avoids the idle-liquidity problem, as it implements the triage for realizations in areas n_2 and n_3. In contrast, under fragmentation, liquid funds stand idle in one country while the other suffers from a financial crisis. If the crisis country could draw on the liquidity of its neighbor it would avoid crisis without infecting it.

The main advantage of an unconditional fragmentation policy is that it avoids contagion from a badly-injured country to mildly-injured country, in those cases where such contagion could not save the badly-injured country, namely for realizations in areas b_3 and a_3. This is not the case under a pooling policy, where these areas are affected by a systemic crisis. Evidently, there is a trade-off between resolving contagion and the idle-liquidity problem.

For realizations in areas b_2 and a_2 the triage prescribes that *contagion does occur* from the badly-injured country to the mildly-injured country when such contagion can save the badly-injured country from financial crisis. This is in spite of the fact that if the mildly injured country keeps its share in the joint liquid fund to itself, it could avoid crisis altogether. This is a dramatic demonstration that, by itself, contagion has no welfare implications. Contagion may well be optimal – in the second-best sense. Neither unconditional fragmentation nor unconditional pooling implements this prescription of the triage.

The analysis of the b_2 and a_2 areas highlights the commitment problem that undermines the practical implementation of the triage. For in these areas, in return for "sacrificing itself" to save its neighbor in an a_2 realization, country A gets the commitment that country B would act in a similar manner in a b_2 realization. Hence, country A trades away a severe crisis (high θ_A realization) for a milder one (low θ_A realization), which is an ex-ante Pareto improvement. Since θ_A and θ_B are observable, one could imagine an implementation of the triage through an international contract between country A and country B, contingent upon (θ_A, θ_B). Given that such a contract would be written between sovereign countries, enforcement problems are likely to be severe, particularly in the "sacrificing" case. This may explain why triage-like policies are not observed in reality. Notice, however, that a contract may be facilitated by an international organization like the IMF, to which both countries hand over their liquidity ex ante, with full control over the allocation of that liquidity, ex post.

6.2. Coordinated unconditional fragmentation

We now consider the case where countries cannot commit to a triage rule, but can agree a coordinated move away from fragmentation (if doing so enhances ex ante welfare). We believe that it is realistic to assume that countries could commit to such an agreement: erecting barriers to capital flows requires structural changes in terms of monitoring, taxation and enforcement that cannot be accomplished in a short period of time. Hence, if a commitment to a certain degree of integration is made, it will be difficult to reverse on a short notice; more so as the incentive to reverse depends on the realization of the shocks θ_A and θ_B – see the triage analysis above. We also generalize the example of Table 1 by considering the entire range between full fragmentation and complete pooling and prove that the coordinated (unconditional) optimum is, indeed, at a corner. We maintain that the total amount of liquidity can be fixed (by setting up a joint liquid fund). The analysis is thus similar to that of the pure fragmentation effect in equation (37), only that we add the effect of changes in country-B's liquidity on country-A's welfare. Due to the fixed $\widehat{\theta}$ (corresponding to a move along horizontal lines in Figure 5, one of which is the dashed $C - A$ line), the expression is actually simpler. Hence (see Appendix for more detail):

$$\begin{aligned}&SW_{\theta_A^*}\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right) + SW_{\theta_B^*}\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right)\\ = \;& \frac{1}{\overline{\theta}}\left[\mu^*\Delta + (1-\mu^*)\, v(\delta)\right]\left[\theta_A^* \frac{\overline{\theta} - \left(\widehat{\theta} - \theta_A^*\right)}{\overline{\theta}} - \left(\widehat{\theta} - \theta_B^*\right)\frac{\theta_B^*}{\overline{\theta}}\right] \\ &-\pi^{RC-A} v\left(\underline{q}\right)\mu_F.\end{aligned} \tag{38}$$

As θ_A^* and θ_B^* increase simultaneously, a systemic crises becomes less likely as it is substituted by regional crises. At the same time the probability of having no crisis also decreases, again due to the increased likelihood of regional crises. This can be seen graphically in Figure 3, where the effects are captured by an expansion of the $RC - B$ and $RC - A$ areas into the SC and NC areas.

The interpretation of equation (38) is similar to that of equation (37) above. Country A benefits from decreasing the probability of crisis at the marginal state θ_A^* by $\left[\overline{\theta} - \left(\widehat{\theta} - \theta_A^*\right)\right]/\overline{\theta}$. At the same time, it suffers from the externality imposed by country B whereby at the marginal state $\left(\widehat{\theta} - \theta_B^*\right)$ the probability of crisis increases by $\theta_B^*/\overline{\theta}$. Finally, the last term in equation (38) captures the changes in credit rationing. Remember that the pool of liquidity available to country A is $\theta_A^* + \eta\left(\widehat{\theta} - \theta_A^* - \theta_B^*\right)$. In the SC area, η equals, on average, to $1/2$. Hence, the increase in both countries' domestic liquidity has no effect on credit availability. At the same time, in the $RC - A$ area, $\eta = 1$ for any realization of (θ_A^*, θ_B^*). It follows that the symmetric increase in both countries' domestic liquidity actually increases the incidence of credit rationing. Since $\widehat{\theta}$ is held constant, any changes in capital gains on the joint fund cancel against changes in the funding costs; for more detail see Appendix.

An immediate implication of equation (38) is that for $\widehat{\theta} > \overline{\theta}$ (and within the relevant range – the shaded area in Figure 5) SW is (weakly) decreasing in fragmentation. To see that, notice that $\left[\overline{\theta} - \left(\widehat{\theta} - \theta_A^*\right) - \left(\widehat{\theta} - \theta_B^*\right)\right] < 0$ for $\widehat{\theta} > \overline{\theta}$ and $\theta_A^* + \theta_B^* < \widehat{\theta}$. Otherwise, Proposition 6 demonstrates that SW is U-shaped (but flattens towards the left side). Both ways, optimal fragmentation has a corner solution. Yet, we provide an analysis of the global maximum: a condition (in terms of a $\widehat{\theta}$ threshold) for which the left end of a fixed-$\widehat{\theta}$ curve is higher than its right end (on the boundary of the relevant area), like the $C - A$ curve in Figure 7.

Proposition 6 *Let*

$$\xi \equiv \frac{\Delta + (1 - \mu^* - \mu_F)\, v\left(\underline{q}\right)}{\mu^* \Delta + (1 - \mu^*)\, v(\delta)} < 1.$$

For a given level of total liquidity, $\widehat{\theta}$, the solution to the optimum, symmetric, fragmentation is pushed to the boundary of the "relevant area" as in Definition 1, namely $\theta_A^* = \theta_B^* = \widehat{\theta}/2$ if

$$\widehat{\theta} < \frac{3}{2}\frac{\xi}{1+\xi}\overline{\theta} \tag{39}$$

hold and full sharing ($\theta_A^* = \theta_B^* = 0$) otherwise. Moreover, $\xi > 0$ for a non-empty parameter set.

Proof see Appendix.

6.3. A welfare analysis of unilateral fragmentation with a binding T

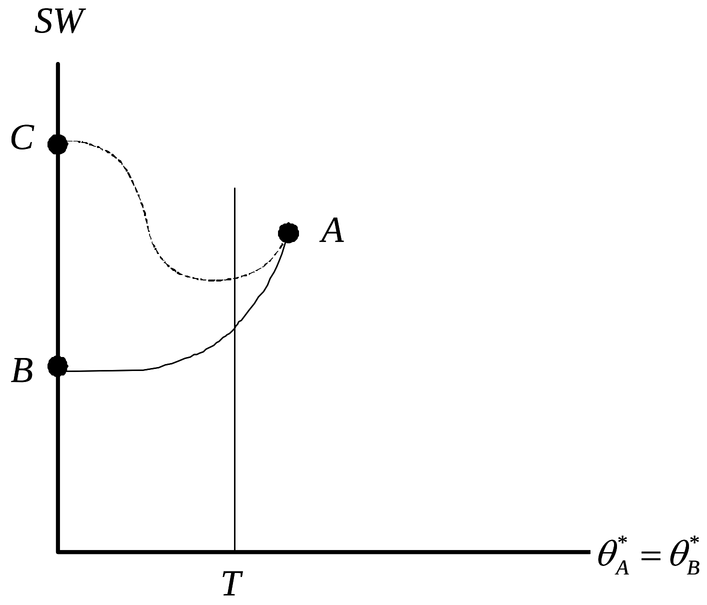
Given our results so far, it is relatively straightforward to prove the next proposition, in a sense our main result, implying that there is "too much fragmentation" in an uncoordinated equilibrium where governments make unilateral fragmentation decisions. By Proposition 4, in absence of coordination the governments would increase local liquidity up to the point where the T-constraint binds (within the relevant area). On the other hand, if countries could coordinate, they would sometimes prefer to pool liquidity, rather than to fragment markets (see Proposition 6). To complete the argument, we need to account for the negative effect of decreased fragmentation on the total amount of liquidity, $\widehat{\theta}$. This is done in the following Proposition.

Proposition 7 *Under coordination countries sometimes choose not to fragment, but to pool liquidity instead.*

Proof see Appendix.

Figure 7

SW along Figure-5 paths, when Proposition 6 is satisfied



To gain a better insight into the argument, consider Figure 7, again. The $B - A$ curve plots social welfare along a symmetric, competitive, equilibrium with $\widehat{\theta} = g(\theta_A^*, \theta_B^*)$. The $C - A$ curve is a member of a family of equal-$\widehat{\theta}$ curves; in this particular case, $\widehat{\theta}$ is at the level where hot money is just fully crowded out (see the corresponding points $C - A$ line in Figure 5). Moreover, the condition in Proposition 7 is satisfied, so that the C-end is higher than the A-end of the curve. Now suppose that the countries agree a coordinated elimination of fragmentation but, also, they agree to keep on contributing to a joint liquid fund what they have previously contributed to their own domestic liquidity. If the T-constraint is close enough to point A, that means moving to the left, along another equal-$\widehat{\theta}$ curve, but slightly lower than the $C - A$ curve. By the continuity of the SW manifold, that would give them a welfare level "a bit" below point C, but still above point A. Hence, there are cases where the uncoordinated equilibrium is at full fragmentation, but the coordinated equilibrium is at zero fragmentation. In that respect, there is "too much fragmentation" in an uncoordinated equilibrium.

7. CONCLUSION

Financial crisis, contagious across countries, is a market failure on a macro scale. Some economists and policy makers have therefore concluded that avoiding contagion by restricting international (short term) capital flows would be socially desirable. We argue that this view is overly simplistic. While market fragmentation ameliorates the contagion problem, it results in an inefficient use of liquidity. We show that while operating unilaterally, countries accumulate domestic liquidity in order to protect themselves from contagion, but in doing so they ignore the negative externality that they exert on their neighbours; hence the old beggar-thy-neighbor problem in economic policy arises.

Our theoretical analysis demonstrates that there is no a-priori reason to believe that the positive welfare effects of fragmentation generically dominate the negative effects. It follows that decentralizing the decision over the amount of domestic liquidity and its mobility may end in a sub-optimal equilibrium. Instead, we suggest a role for policy coordination. In its crudest form this could be through coordinating ex ante whether to restrict capital flows. Ideally countries should coordinate an a more refined arrangement, whereby the allocation of liquidity is not carried out by market forces, but through a contingency rule, similar to the triage in emergency medicine. To overcome commitment problems, we suggest a role for an independent, international organization (say, the IMF) that can take a decision to ex-post "sacrifice" one country in order to save another.

Our analysis considers two instruments only: fragmentation and liquidity injection. It calls for further research into the operation of other instruments that may relieve the problem of liquidity under-provision, yet avoid a more severe idle-funds or crowding-out problem.

8. APPENDIX

Proof of Lemma 1. It follows from the discussion of (4) that the solution of the contracting problem is the minimal β within the problem's feasible set. The graph of (PC) is a downwards-sloping line in the (r, β) space; (IC) is represented by the area above an upwards-sloping line in the same space. Hence, the minimal point that satisfies both (IC) and (PC) is the intersection point between the two lines given by the binding (PC) and (IC). That point is defined by equation (5). If $b \leq 1$, then (5) also defines the optimal contract; if $b > 1$ the feasible set is empty and the entrepreneur cannot obtain any funding. ■

Proof of Lemma 2. Let $\underline{\underline{q}}$ be a fire-sale price that satisfies both $b = 1$ and and $c|_{\underline{w}} = \rho_1 \underline{w}$. By (IC) $r = y$ when $\beta = 1$, which implies that $c|_{\underline{w}} = \pi y$. Hence, $\underline{\underline{q}}$ must satisfy

$$\underline{w}\frac{\delta}{\underline{\underline{q}}} = \pi y.$$

Substituting this expression into the binding (PC) at the contract $\beta = 1, r = y$ and solving for $\underline{\underline{q}}$ yields

$$\underline{\underline{q}} = \frac{-\pi y + \sqrt{(\pi y)^2 + (1-\pi)\,\delta}}{(1-\pi)}.$$

Define $z(\underline{w}) \equiv \underline{q} - \underline{\underline{q}}$, where

$$z(\underline{w}) = \frac{1}{(1-\pi)}\left[\frac{\pi y}{2} + \sqrt{\left(\frac{\pi y}{2}\right)^2 + (1-\pi)\,\delta\,(1-\underline{w})} - \sqrt{(\pi y)^2 + (1-\pi)\,\delta}\right].$$

It is easy to verify that $z\left(\underline{w}=0\right)>0$ and $z\left(\underline{w}=1\right)<0$. Moreover, $z\left(\underline{w}\right)$ is continuous and strictly decreasing in $\underline{w}$. It follows that there is a unique $\underline{\underline{w}}$ such that $z\left(\underline{\underline{w}}\right)=0$ and $z\left(\underline{w}<\underline{\underline{w}}\right)>0$ (i.e., $\underline{\underline{q}}<\underline{q}$). Finally, $\underline{\underline{q}}<\underline{q}$ implies $c|_{\underline{w}}>\rho_1\underline{w}$ since $\underline{q}$ is decreasing in $\underline{w}$ and ρ_1 decreasing in $\underline{q}$. ■

Proof of Proposition 1. Substituting $w_a=1$ into the market clearing condition (8) and evaluating at $q=\delta,\mu=1$ yields

$$F-\theta\delta\left(1-\pi\right)b\left(\delta\right)\geq 0.$$

When this condition holds, there is enough liquidity to absorb fire sales and fund all the entrepreneurs ($\mu=1$), even when the price of investment goods is bid up to their fair value $q=\delta$. When this condition fails to hold, and since $qb\left(q\right)$ is decreasing in q, the market-clearing condition (8) cannot hold with $\mu=1$ for any other price in the feasible range $\left[\underline{q},\delta\right]$. Hence, for any θ above the critical point, $\theta>\frac{F}{\delta(1-\pi)b(\delta)}$, μ must drop below 1 to satisfy (8), i.e., credit rationing is the unique equilibrium.

For the fire-sale price to drop below δ it must be that there is insufficient liquidity to fund all entrepreneurs and absorb all fire-sales. Consider the highest possible liquidity demand (that is liquidity demand from fire sales is highest (i.e., when $q=\underline{q}$) and all entrepreneurs are funded). We would then have to have

$$F-\theta\underline{q}\left(1-\pi\right)\leq 0.$$

It is clear that when this condition does not hold, the fire-sale price would be bid up to $q=\delta$, and since $qb\left(q\right)$ is decreasing in q, there must then be an equilibrium with excess supply of liquidity. Hence, for any $\theta<\frac{F}{\underline{q}(1-\pi)}$, excess supply of liquidity is the unique equilibrium.

Since $\underline{q}>\delta b\left(\delta\right)$, we have $\frac{F}{\underline{q}(1-\pi)}<\frac{F}{\delta(1-\pi)b(\delta)}$. Consider now θ between these thresholds. From the above discussion we know that at $q=\delta$, and $\mu=1$ there is excess liquidity, so this is an equilibrium. Alternatively, we know that for $q=\underline{q}$, there are insufficient funds to finance all entrepreneurs, so there is an equilibrium with $\mu<1$. Finally, there are values of $q\in\left(\underline{q},\delta\right)$ such that (8) holds with equality at $\mu=1$, i.e., there is just enough liquidity (but no excess liquidity) to finance all entrepreneurs at a price $q<\delta$.

Substituting $q=\underline{q}$ and $b\left(\underline{q}\right)=1$ into the market-clearing condition (8) we can solve for the fraction of entrepreneurs who are credit rationed (equation (9)). ■

Derivation of $SW\left(\theta^*\right)$ in one-country benchmark and equation (16). Let $W\left(\theta\right)$ be expected consumption, conditional on the macro state, θ, but unconditional on the individual capital shock, and gross of the cost of funding liquidity.

If there is no crisis ($\theta\leq\theta^*$), there is a fraction θ of entrepreneurs who need external finance. A fraction $1-\pi$ of them will be subject to a liquidity shock and thus lose

$b(\delta)$ in collateral. Their expected payoff is therefore $\pi(2y-r)+(1-\pi)2y(1-b(\delta))$. Substituting r by its value from the binding (PC) into this expression yields a payoff

$$\underline{w}+2y-1-(1-\pi)b(\delta)(2y-\delta).$$

Those entrepreneurs who are capital-rich have a payoff

$$w^n+2y-1.$$

Note that investment goods trade at their fair value $q=\delta$ and therefore no taxation / redistribution of capital gains is necessary. Conditional on being out of crisis, the expected payoff is therefore

$$\begin{aligned} W(\theta \mid \theta \leq \theta^*) &= \theta\left[\underline{w}+2y-1-(1-\pi)b(\delta)(2y-\delta)\right] \\ &\quad +(1-\theta)(w^n+2y-1). \end{aligned}$$

Substituting w_a from (1) then yields

$$W(\theta \mid \theta \leq \theta^*) = 2y-\theta(1-\pi)b(\delta)(2y-\delta). \tag{40}$$

Intuitively, the expression gives the country's potential output, $2y$, less the deadweight loss of pre-mature liquidations.

Consider now payoffs during crisis. Conditional on being capital poor, an entrepreneur is credit rationed with probability $1-\mu$. In that case he invests his endowment at the market rate of return $\frac{\delta}{\underline{q}}$. His payoff is therefore

$$\underline{w}\frac{\delta}{\underline{q}}.$$

If the entrepreneur has access to external financing, his payoff is the project payoff $2y$ minus the cost of capital $(1-\underline{w})\frac{\delta}{\underline{q}}$, minus the deadweight loss of early liquidation $(1-\pi)\left(2y-\underline{q}\right)$:

$$2y-(1-\underline{w})\frac{\delta}{\underline{q}}-(1-\pi)\left(2y-\underline{q}\right).$$

Finally, a capital-rich entrepreneur earns

$$w^n\frac{\delta}{\underline{q}}+2y-\frac{\delta}{\underline{q}}.$$

Moreover, there is a capital gain on the liquidity F provided: $F\left(\frac{\delta}{\underline{q}}-1\right)$. If the government provides the liquidity, the capital gains are distributed back as a lump sum to all entrepreneurs. The expected payoff is therefore

$$\begin{aligned} W(\theta \mid \theta > \theta^*) &= \theta\left\{\mu\left[2y-(1-\underline{w})\frac{\delta}{\underline{q}}-(1-\pi)\left(2y-\underline{q}\right)\right]+(1-\mu)\underline{w}\frac{\delta}{\underline{q}}\right\} \\ &\quad +(1-\theta)\left(w^n\frac{\delta}{\underline{q}}+2y-\frac{\delta}{\underline{q}}\right) \\ &\quad +F\left(\frac{\delta}{\underline{q}}-1\right). \end{aligned}$$

Substituting w_a into the expression and simplifying yields

$$W(\theta \mid \theta > \theta^*) = 2y - \theta\left[\mu(1-\pi)(2y-\underline{q}) + (1-\mu)\left(2y - \frac{\delta}{\underline{q}}\right)\right] + F\left(\frac{\delta}{\underline{q}} - 1\right).$$

Using the fact that μ is a function of F according to (9), the above expression can be simplified to

$$W(\theta \mid \theta > \theta^*) = 2y - \theta\left[\mu(1-\pi)(2y-\delta) + (1-\mu)(2y-1)\right]. \tag{41}$$

Once the capital gains of rich entrepreneurs are netted out against the higher cost of capital of capital poor entrepreneurs, expected payoffs can be interpreted as the payoff $2y$ minus the social cost of early liquidation of the externally funded projects $\theta\mu(1-\pi)(2y-\delta)$, minus the social loss of credit rationing $\theta(1-\mu)(2y-1)$.

Since liquidity could have been invested in the illiquid technology at rate ρ_0, the opportunity cost is $F(\rho_0 - 1)$. This must be subtracted from the entrepreneurs' payoffs to arrive at overall social welfare.

Given θ^* we then take expectations over $W(\theta)$ and get expected social welfare as a function of θ^*:

$$\begin{aligned} SW(\theta^*) &= \int_0^{\theta^*} \left[2y - \theta(1-\pi)b(\delta)(2y-\delta)\right]\frac{1}{\overline{\theta}}d\theta \\ &+ \int_{\theta^*}^{\overline{\theta}} \left\{2y - \theta\left[\mu(1-\pi)(2y-\delta) + (1-\mu)(2y-1)\right]\right\}\frac{1}{\overline{\theta}}d\theta \\ &- F(\rho_0 - 1). \end{aligned}$$

Solving the integration and using the definitions (13) - (15) allows us to rewrite social welfare as

$$\begin{aligned} SW(\theta^*) &= 2y - (1-\pi)b(\delta)(2y-\delta)\frac{\theta^*}{\overline{\theta}}\frac{\theta^*}{2} \\ &- \left[2y - 1 - v(\underline{q})\mu_\theta\right]\left(1 - \frac{\theta^*}{\overline{\theta}}\right)\frac{\overline{\theta}+\theta^*}{2} + v(\underline{q})\mu_F\theta^*\left(1 - \frac{\theta^*}{\overline{\theta}}\right) \\ &- F(\rho_0 - 1). \end{aligned} \tag{42}$$

By way of interpretation, note that $\frac{\theta^*}{\overline{\theta}}\frac{\theta^*}{2}$ is the expectation of θ conditional on being out of crisis ($\theta \leq \theta^*$) and $\left(1 - \frac{\theta^*}{\overline{\theta}}\right)\frac{\overline{\theta}+\theta^*}{2}$ is expectation of θ conditional on being in crisis ($\theta > \theta^*$).

Substituting F as a function of θ^* given from (10), the derivative follows directly from (42). ∎

Proof of Proposition 3. In the case of a convex SW a sufficient condition for the corner solution $\theta^* = \overline{\theta}$ is $SW'(0) > (\rho_0 - 1) F_{\theta^*}$. Using (16), this can be written as

$$v\left(\underline{q}\right)\mu_F > (\rho_0 - 1) F_{\theta^*}. \tag{43}$$

It follows from assumption (A1) that

$$v\left(\underline{q}\right)\mu_\theta \geq \left(\frac{\delta}{\underline{q}} - 1\right),$$

or

$$v\left(\underline{q}\right)\mu_F \geq \left(\frac{\delta}{\underline{q}} - 1\right) F_{\theta^*},$$

or

$$Rv\left(\underline{q}\right)\mu_F \geq (\rho_0 - 1) F_{\theta^*}.$$

Since $R < 1$, the last line implies (43).

In the case of a concave SW the condition for a corner solution at $\theta^* = \overline{\theta}$ is $SW'\left(\theta^* = \overline{\theta}\right) > (\rho_0 - 1) F_{\theta^*}$ Using (16) this can be written as

$$v(\delta) - \mu^* v\left(\underline{q}\right) > (\rho_0 - 1) F_{\theta^*}. \tag{44}$$

It follows from (13), (14), and (16)that

$$v(\delta) - \mu^* v\left(\underline{q}\right) > (1 - \pi)\left[1 - b(\delta)\right](2y - \delta).$$

Moreover, we know that $\underline{q} > \delta b(\delta)$, which can be written as

$$1 - b(\delta) > \frac{\frac{\delta}{\underline{q}} - 1}{\frac{\delta}{\underline{q}}}.$$

It also follows from $y > \delta$ that $(2y - \delta) > \delta$. Using both we conclude that

$$v(\delta) - \mu^* v\left(\underline{q}\right) > (1 - \pi)\,\underline{q}\left(\frac{\delta}{\underline{q}} - 1\right).$$

Using $\underline{q} > \delta b(\delta)$ once again and $\left(\frac{\delta}{\underline{q}} - 1\right) > (\rho_0 - 1)$ we conclude that $(1 - \pi)\,\underline{q}\left(\frac{\delta}{\underline{q}} - 1\right) > (\rho_0 - 1) F_{\theta^*}$ and thus (44) holds. ■

Derivation of $SW\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right)$ for two-country economy. We calculate welfare of country A for each of the four equilibrium regimes, denoted by W^{NC}, W^{RC-A}, W^{RC-B} and W^{SC}. Consider first the case when neither country is in crisis. In this simple case $q_i = \delta$ in both countries and no country suffers from credit rationing. Country-A welfare is given by the same expression as in the single country case before (see (40)):

$$W^{NC} = 2y - \theta_A (1 - \pi) b(\delta)(2y - \delta). \tag{45}$$

In case of regional crises, if the crisis hits country B only $(RC-B)$, welfare in country A is not affected

$$W^{RC-B} = W^{NC}. \tag{46}$$

However, if country A is in crisis, a share $1-\mu_A$ of its companies suffer from credit rationing. In this case, both domestic and international liquidity make speculative capital gains, but domestic gains are distributed back. Hence,

$$W^{RC-A} = 2y - \theta_A \left[\mu_A (1-\pi)(2y - \underline{q}) + (1-\mu_A)\left(2y - \frac{\delta}{\underline{q}}\right)\right] + F_A \left(\frac{\delta}{\underline{q}} - 1\right).$$

Using the fact that F_A can be expressed as a function of μ_A using (24), we can write

$$W^{RC-A} = 2y - \theta_A \left[\mu_A (1-\pi)(2y-\delta) + (1-\mu_A)(2y-1)\right] - F\left(\frac{\delta}{\underline{q}} - 1\right). \tag{47}$$

Namely, after redistributing domestic capital gains, the country "pays" capital gains to the international speculators only. Welfare equals $2y$ less the social cost of liquidation less capital gains of international speculators.

Welfare of country A in the SC regime is analogous to (47) except that country A only uses a fraction η of the international funds.

$$W^{SC} = 2y - \theta_A \left[\mu_A (1-\pi)(2y-\delta) + (1-\mu_A)(2y-1)\right] - \eta F\left(\frac{\delta}{\underline{q}} - 1\right), \tag{48}$$

and symmetrically for country B. Notice that η depends on θ_B and θ_B^*.

In addition domestic liquidity carries an opportunity cost, so that welfare is reduced by $(\rho_0 - 1) F_A$. Moreover, for the general case, we need to calculate the costs and payoffs to the internationally mobile liquidity (hot money). If the latter is provided by competitive speculators, the payoff on the fund is just zero, since supply is determined by a break-even condition (so treating hot money as being supplied through a government-owned joint fund does not affect the welfare accounting). Call the expected net payoff of the joint liquid fund

$$L\left(\theta_A^*, \theta_B^*, \widehat{\theta}\right) = \left(\pi^{SC} + \pi^{RC-A} + \pi^{RC-B}\right)\left(\frac{\delta}{\underline{q}} - 1\right) F - (\rho_0 - 1) F. \tag{49}$$

Each country owns one half of this fund. Overall welfare of country A is then given by integrating the welfare functions (45) - (48) over the relevant regions of θ_A and θ_B, subtracting the opportunity cost of domestic liquidity and adding the net payoff to country

A's share in the joint liquid fund:

$$SW\left(\theta_A^*,\theta_B^*,\widehat{\theta}\right) = \int_0^{\theta_A^*}\int_0^{\widehat{\theta}-\theta_A^*} W^{NC} d\theta_B d\theta_A + \int_{\theta_A^*}^{\widehat{\theta}-\theta_B^*}\int_0^{\widehat{\theta}-\theta_A} W^{NC} d\theta_B d\theta_A \quad (50)$$
$$+\int_0^{\theta_A^*}\int_{\widehat{\theta}-\theta_A^*}^{\overline{\theta}} W^{RC-B} d\theta_B d\theta_A$$
$$+\int_{\widehat{\theta}-\theta_B^*}^{\overline{\theta}}\int_0^{\widehat{\theta}-\theta_A} W^{RC-A} d\theta_B d\theta_A$$
$$+\int_{\theta_A^*}^{\widehat{\theta}-\theta_B^*}\int_{\widehat{\theta}-\theta_A}^{\overline{\theta}} W^{SC} d\theta_B d + \int_{\widehat{\theta}-\theta_B^*}^{\overline{\theta}}\int_{\theta_B^*}^{\overline{\theta}} W^{SC} d\theta_B d\theta_A$$
$$-(\rho_0-1)F_A + \frac{1}{2}L\left(\theta_A^*,\theta_B^*,\widehat{\theta}\right).$$

Note here that when hot money is supplied competitively, $\widehat{\theta}$ is determined as an implicit function of θ_A^* so as to ensure $L\equiv 0$. Hence, for $\widehat{\theta}=g(\theta_A^*,\theta_B^*)$, $\frac{dL\left(\theta_A^*,\theta_B^*,\widehat{\theta}\right)}{d\theta_A^*}=0$. Calculating the partial derivative of $SW\left(\theta_A^*,\theta_B^*,\widehat{\theta}\right)$ with respect to θ_A^* and applying definitions (14)-(15) and (17)-(20) yields (37). ■

Derivation of $\overline{\eta},\overline{\eta}_{\theta_A^*}$, $\overline{\eta}_{\theta_B^*}$ and $\overline{\eta}_{\widehat{\theta}}$. The η function is defined on the SC equilibrium regime only:

$$\eta:\left\{(\theta_A,\theta_B)\,|\,\theta_A\in\left[\theta_A^*,\overline{\theta}\right],\theta_B\in\left[\theta_B^*,\overline{\theta}\right],\theta_A+\theta_B>\widehat{\theta}\right\}\rightarrow[0,1].$$

To analyze the allocation of the cost of funding to the two countries we need to take expectations over all realizations within the SC regime, i.e., denoting $\overline{\eta}=E^{SC}(\eta)$, we calculate

$$\overline{\eta} = \frac{1}{\overline{\theta}^2}\int_{\theta_A^*}^{\widehat{\theta}-\theta_B^*}\int_{\widehat{\theta}-\theta_A}^{\overline{\theta}} \eta\, d\theta_B d\theta_A \quad (51)$$
$$+\frac{1}{\overline{\theta}^2}\int_{\widehat{\theta}-\theta_B^*}^{\overline{\theta}}\int_{\theta_B^*}^{\overline{\theta}} \eta\, d\theta_B d\theta_A. \quad (52)$$

Doing the integration in (51) and simplifying yields

$$\overline{\eta} = -\overline{\theta}\left(\theta_A^*-\theta_B^*\right)\ln\left(2\overline{\theta}-\theta_A^*-\theta_B^*\right) \quad (53)$$
$$+\frac{1}{2}\left[\left(\theta_A^*\right)^2-\left(\theta_B^*\right)^2\right]\ln\left(2\overline{\theta}-\theta_A^*-\theta_B^*\right)$$
$$-\frac{1}{2}\left(\overline{\theta}-\theta_A^*\right)^2\ln\left(\overline{\theta}-\theta_A^*\right)$$
$$+\frac{1}{2}\left(\overline{\theta}-\theta_B^*\right)^2\ln\left(\overline{\theta}-\theta_B^*\right)$$
$$+\frac{1}{2}\pi^{SC}.$$

In what follows we will be interested in properties of the expected share of liquidity at the symmetry points $\theta_A^* = \theta_B^*$. Define the partial derivatives of $\overline{\eta}$ with respect to changes in domestic and international liquidity, evaluated at the symmetry points, by

$$\overline{\eta}_{\theta_A^*} \equiv \left.\frac{\partial \overline{\eta}}{\partial \theta_A^*}\right|_{\theta_A^*=\theta_B^*}, \overline{\eta}_{\theta_B^*} \equiv \left.\frac{\partial \overline{\eta}}{\partial \theta_B^*}\right|_{\theta_A^*=\theta_B^*}, \text{ and } \overline{\eta}_{\widehat{\theta}} = \left.\frac{\partial \overline{\eta}}{\partial \widehat{\theta}}\right|_{\theta_A^*=\theta_B^*}.$$

At the symmetry points $\theta_A^* = \theta_B^*$, the top four lines of (53) vanish, so that:

$$\overline{\eta} = \frac{1}{2}\pi^{SC}. \tag{54}$$

Taking the partial derivative of the expression in (53) with respect to θ_A^* yields

$$\overline{\eta}_{\theta_A^*} = \frac{1}{\overline{\theta}^2}\left[-\left(\overline{\theta} - \theta_A^*\right)\ln(2) + \frac{1}{2}\left(\widehat{\theta} - 2\theta_A^*\right)\right]. \tag{55}$$

Next, again from (53) we calculate

$$\overline{\eta}_{\theta_B^*} = \frac{1}{\overline{\theta}^2}\left[\left(\overline{\theta} - \theta_A^*\right)\ln(2) - \frac{1}{2}\left(2\overline{\theta} - \widehat{\theta}\right)\right]. \tag{56}$$

Finally, take the partial derivative of (53) with respect to $\widehat{\theta}$ to get

$$\overline{\eta}_{\widehat{\theta}} = -\frac{1}{2\overline{\theta}^2}\left(\widehat{\theta} - 2\theta_A^*\right) \tag{57}$$

■

Proof of Proposition 4. Consider first the "liquidity" effect. Using (50) we can calculate the derivative at the symmetry point $\theta_A^* = \theta_B^* = \theta_d^*$:

$$\begin{aligned} SW_{\widehat{\theta}}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right) = {} & \frac{1}{\overline{\theta}^2}\left[\frac{1}{2}\widehat{\theta}\left(\widehat{\theta} - 2\theta_d^*\right) + \theta_d^*\left(\widehat{\theta} - \theta_d^*\right)\right]\left[\mu^*\Delta + (1-\mu^*)\,v(\delta)\right] \quad (58) \\ & + \left[\frac{1}{2}\pi^{SC} + \pi^{RC-A}\right]\mu_F v\left(\underline{q}\right) \\ & - \left[\frac{1}{2}\pi^{SC} + \pi^{RC-A} - \frac{1}{2\overline{\theta}^2}\widehat{\theta}\left(\widehat{\theta} - 2\theta_d^*\right)\right]\left(\frac{\delta}{\underline{q}} - 1\right)F_{\theta^*} \\ & + \frac{1}{2}L_{\widehat{\theta}}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right). \quad (59) \end{aligned}$$

Using equations (32) and (30) we can write equation (58) as

$$\begin{aligned} SW_{\widehat{\theta}}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right) = {} & \frac{1}{2}\left[\left(\frac{\widehat{\theta}}{\overline{\theta}}\right)^2 - 2\left(\frac{\theta_d^*}{\overline{\theta}}\right)^2\right]\left[\mu^*\Delta + (1-\mu^*)\,v(\delta)\right] \\ & + \frac{1}{2}\left\{1 - \frac{1}{2}\left[\left(\frac{\widehat{\theta}}{\overline{\theta}}\right)^2 - 2\left(\frac{\theta_d^*}{\overline{\theta}}\right)^2\right]\right\}\mu_F v\left(\underline{q}\right) \\ & - \frac{1}{2}\left[R - \frac{1}{\overline{\theta}^2}\widehat{\theta}\left(\widehat{\theta} - 2\theta_d^*\right)\right]\left(\frac{\delta}{\underline{q}} - 1\right)F_{\theta^*} \\ & + \frac{1}{2}L_{\widehat{\theta}}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right). \end{aligned}$$

Since $L_{\theta_A^*}\left(\theta_d^*,\theta_d^*,\widehat{\theta}\right)+\frac{\partial g\left(\theta_A^*,\theta_B^*\right)}{\partial\theta_A^*}L_{\widehat{\theta}}\left(\theta_d^*,\theta_d^*,\widehat{\theta}\right)=0$, we drop the corresponding terms when evaluating the sign of both the liquidity and fragmentation effects. Hence,

$$SW_{\widehat{\theta}}\left(\theta_d^*,\theta_d^*,\widehat{\theta}\right)-\frac{1}{2}L_{\widehat{\theta}}\left(\theta_d^*,\theta_d^*,\widehat{\theta}\right)>\frac{1}{2}\left\{\begin{array}{c}\frac{1}{2}\left[\left(\frac{\widehat{\theta}}{\overline{\theta}}\right)^2-2\left(\frac{\theta_d^*}{\overline{\theta}}\right)^2\right]\left[\mu^*\Delta+(1-\mu^*)\,v\left(\delta\right)\right]\\+\left(1-\frac{1}{2}\left[\left(\frac{\widehat{\theta}}{\overline{\theta}}\right)^2-2\left(\frac{\theta_d^*}{\overline{\theta}}\right)^2\right]\right)\mu_F v\left(\underline{q}\right)\\-\left(\rho_0-1\right)F_{\theta^*}\end{array}\right\}.$$

The expression inside the { } is the single-country SW' at the point where the probability of crisis is equal to $\frac{1}{2}\left[\left(\frac{\widehat{\theta}}{\overline{\theta}}\right)^2-2\left(\frac{\theta_B^*}{\overline{\theta}}\right)^2\right]$. It follows from $\pi^C\in(0,1)$ that $\frac{1}{2}\left[\left(\frac{\widehat{\theta}}{\overline{\theta}}\right)^2-2\left(\frac{\theta_B^*}{\overline{\theta}}\right)^2\right]\in(0,1)$. Hence, we can apply Proposition 3 to conclude that the liquidity effect is positive.

Given Lemma 5 and a positive liquidity effect, it is sufficient to show that the "fragmentation" effect is positive:

$$SW_{\theta_A^*}\left(\theta_d^*,\theta_d^*,\widehat{\theta}\right)>0.$$

Using equation (30) we can rewrite the fragmentation effect (37) as

$$\begin{aligned}SW_{\theta_A^*}\left(\theta_d^*,\theta_d^*,\widehat{\theta}\right)-\frac{1}{2}L_{\theta_A^*}\left(\theta_d^*,\theta_d^*,\widehat{\theta}\right)&=\pi^{RC-A}\left[\mu^*\Delta+(1-\mu^*)\,v\left(\delta\right)-\left(\frac{\delta}{\underline{q}}-1\right)F_{\theta^*}\right]\\&\quad+\left[\frac{1}{2}\pi^{SC}+\left(\widehat{\theta}-2\theta_d^*\right)\overline{\eta}_{\theta_A^*}\right]\left[\mu_F v\left(\underline{q}\right)-\left(\frac{\delta}{\underline{q}}-1\right)F_{\theta^*}\right].\end{aligned}$$

Proposition 3 already shows that the expressions in curly brackets are positive. It is left to be shown that the expression in square brackets is positive:

$$\begin{aligned}\frac{1}{2}\pi^{SC}+\left(\widehat{\theta}-2\theta_d^*\right)\overline{\eta}_{\theta_A^*}&=\frac{1}{2\overline{\theta}^2}\left\{\begin{array}{c}\left(\overline{\theta}-\theta_d^*\right)\left(\overline{\theta}-\widehat{\theta}+\theta_d^*\right)\\+\left(\widehat{\theta}-2\theta_d^*\right)\left[\frac{1}{2}\left(\widehat{\theta}-2\theta_d^*\right)-\left(2\ln\left(2\right)-1\right)\left(\overline{\theta}-\theta_d^*\right)\right]\end{array}\right\}\\&>\frac{1}{2\overline{\theta}^2}\left(\overline{\theta}-\widehat{\theta}+\theta_d^*\right)\left(\overline{\theta}-\frac{1}{2}\widehat{\theta}\right)>0.\end{aligned}$$

At the last step we use the fact that $\left[2\ln\left(2\right)-1\right]<\frac{1}{2}$. ■

Proof of Proposition 5. If there is enough liquidity to keep both countries out of crisis ($\theta_A+\theta_B\leq\widehat{\theta}$), it is obvious that this is ex post optimal to do so. Suppose the realization of shocks is such that this is not possible ($\theta_A+\theta_B>\widehat{\theta}$). We then need to check if and when it is better to contain the crisis regionally, or have a systemic crisis. For the purpose of welfare accounting we assume that all capital gains on liquidity are redistributed back to either country. Welfare of country A is given by equations (45)-(48) and analogously for country B by replacing the country index. We then apply definitions (14)-(15) and (17)-(20) and calculate the joint welfare of countries A and B under the various regimes:

If both countries are in crisis total welfare is given by

$$W_{A+B}^{SC}=4y-\left(\theta_A+\theta_B\right)\left[2y-1-v\left(\underline{q}\right)\mu_\theta\right]+\left(F_A+F_B\right)v\left(\underline{q}\right)\mu_\theta.$$

Obviously, it is optimal not to leave any liquidity idle, so that $F_A + F_B = \widehat{\theta} F_{\theta^*}$. Note that the ex post sharing scheme does not affect overall welfare, conditional on both countries experiencing a crisis.

Suppose liquidity is allocated such that one country (say A) is in crisis, and the other is not. Welfare is then given by

$$\begin{aligned} W_{A+B}^{RC-A} &= 4y - \theta_A \left[2y - 1 - v\left(\underline{q}\right)\mu_\theta\right] \\ &\quad -\theta_B\left(1-\pi\right)b\left(\delta\right)\left(2y-\delta\right) + v\left(\underline{q}\right)\mu_\theta F_A. \end{aligned} \tag{60}$$

Again, it will be optimal to allocate all the liquidity to A that is left after keeping country B out of crisis. Hence, $F_A = \left(\widehat{\theta} - \theta_B\right) F_{\theta^*}$.

It follows that it is preferable to keep one country out of crisis, if

$$4y - \left[2y - 1 - v\left(\underline{q}\right)\mu_\theta\right]\theta_A - \left(1-\pi\right)b\left(\delta\right)\left(2y-\delta\right)\theta_B + v\left(\underline{q}\right)\mu_F\left(\widehat{\theta} - \theta_B\right) \tag{61}$$

$$\geq 4y - \left[2y - 1 - v\left(\underline{q}\right)\mu_\theta\right]\left(\theta_A + \theta_B\right) + v\left(\underline{q}\right)\mu_F\widehat{\theta}. \tag{62}$$

This inequality can be rewritten as

$$2y - 1 - v\left(\underline{q}\right)\mu_\theta - \left(1-\pi\right)b\left(\delta\right)\left(2y-\delta\right) - v\left(\underline{q}\right)\mu_F \geq 0,$$

which amounts to

$$\Delta + v\left(\underline{q}\right)\left(1-\mu^*\right) \geq 0,$$

which is always true under our parametric assumptions. Hence, a systemic crisis should always be avoided, if that is feasible.

Regarding the choice which country should be allowed to remain out of crisis, we can apply (60) to a regional crisis in country B and then compare. Country A should be kept out of crisis at the expense of country B being allowed to move into crisis, if

$$\begin{aligned} & 4y - \left[2y - 1 - v\left(\underline{q}\right)\mu_\theta\right]\theta_A - \left(1-\pi\right)b\left(\delta\right)\left(2y-\delta\right)\theta_B + v\left(\underline{q}\right)\mu_F\left(\widehat{\theta} - \theta_B\right) \\ < \; & 4y - \left[2y - 1 - v\left(\underline{q}\right)\mu_\theta\right]\theta_B - \left(1-\pi\right)b\left(\delta\right)\left(2y-\delta\right)\theta_A + v\left(\underline{q}\right)\mu_F\left(\widehat{\theta} - \theta_A\right) \end{aligned}$$

which boils down to

$$\theta_B < \theta_A.$$

The optimal allocation is thus given by combining the optimality conditions with the feasibility conditions. ■

Derivation of equation 38. First we need to derive the derivative of social welfare with respect to a joint increase in θ_A^* and θ_B^*. The funding cost of local liquidity is attributed to the country providing it. Capital gains on local liquidity are distributed back to locals. $SW_{\theta_A^*}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right)$ is given by (37). The derivative of country A social welfare with respect to an increase in θ_B^* can be calculated as:

$$
\begin{aligned}
SW_{\theta_B^*}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right) &= -\left[\mu^* \Delta + (1-\mu^*) v(\delta)\right] \frac{1}{\overline{\theta}^2} \left[\theta_d^* \left(\widehat{\theta} - \theta_d^*\right)\right] \\
&\quad - \left[\frac{1}{2}\pi^{SC} + \pi^{RC-A} - \left(\widehat{\theta} - 2\theta_d^*\right) \left(\eta_{\theta_B^*} + \frac{\overline{\theta} - \widehat{\theta} + \theta_d^*}{\overline{\theta}^2}\right)\right] \mu_F v\left(\underline{q}\right) \\
&\quad + \left[\frac{1}{2}\pi^{SC} + \pi^{RC-A} - \left(\eta_{\theta_B^*} + \frac{\overline{\theta} - \widehat{\theta} + 2\theta_d^*}{\overline{\theta}^2}\right) \left(\widehat{\theta} - 2\theta_d^*\right)\right] \left(\frac{\delta}{\underline{q}} - 1\right) F_{\theta^*} \\
&\quad + \frac{1}{2} L_{\theta_B^*}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right).
\end{aligned}
$$

Regarding the payoff L to the joint liquid fund, note that $F = \theta^* F_{\theta^*}$ and $\theta^* = \left(\widehat{\theta} - 2\theta_d^*\right)$. Substituting this into (49) and evaluating the derivative at the point $\theta_A^* = \theta_B^* = \theta_d^*$ yields

$$
\begin{aligned}
\frac{1}{2}\left[L_{\theta_A^*}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right) + L_{\theta_B^*}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right)\right] &= \frac{\theta_d^*}{\overline{\theta}^2}\left(\widehat{\theta} - 2\theta_d^*\right)\left(\frac{\delta}{\underline{q}} - 1\right) F_{\theta^*} \\
&\quad - \left(\pi^{SC} + 2\pi^{RC-A}\right)\left(\frac{\delta}{\underline{q}} - 1\right) F_{\theta^*} \\
&\quad + (\rho_0 - 1) F_{\theta^*}.
\end{aligned}
$$

Adding up and simplifying yields

$$
\begin{aligned}
& SW_{\theta_A^*}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right) + SW_{\theta_B^*}\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right) \\
= \; & \left[\mu^* \Delta + (1-\mu^*) v(\delta)\right] \frac{\theta_d^*}{\overline{\theta}^2}\left[\overline{\theta} - \widehat{\theta} + \theta_d^* - \left(\widehat{\theta} - \theta_d^*\right)\right] \\
& - \pi^{RC-A} \mu_F v\left(\underline{q}\right).
\end{aligned}
$$

■

Proof of Proposition 6. Using the definition

$$
\xi \equiv \frac{\Delta + v\left(\underline{q}\right)(1 - \mu^* - \mu_F)}{\mu^* \Delta + (1-\mu^*) v(\delta)},
$$

we immediately can rewrite equation (38) as

$$
\left(\mu^* \Delta + (1-\mu^*) v(\delta)\right) \frac{\theta_d^*}{\overline{\theta}^2}\left(\xi\left(\overline{\theta} - \widehat{\theta} + \theta_d^*\right) - \left(\widehat{\theta} - \theta_d^*\right)\right).
$$

It follows from the previous lemma that there is a critical point at which the optimal level of fragmentation "jumps" from zero to $\widehat{\theta}/2$. To find out the critical level at which such a jump increases $SW\left(\theta_d^*, \theta_d^*, \widehat{\theta}\right)$ we solve

$$
\int_0^{\widehat{\theta}/2} \theta_d^* \left[\xi\left(\overline{\theta} - \widehat{\theta} + \theta_d^*\right) - \left(\widehat{\theta} - \theta_d^*\right)\right] d\theta_d^* = 0.
$$

■

Proof of Proposition 7. Consider a situation where the T-constraint is binding (see Figure 5), so total liquidity, $\widehat{\theta}$, is determined at the intersection of the T constraint and the $A-B$ curve. It can be shown that along the $A-B$ curve in Figure 5, SW is increasing – see the corresponding $A-B$ curve in Figure 7[11]. Assume, also, that the condition in Proposition 6 is satisfied, so that welfare along an $A-C$ line is U-shaped (flattening towards the left) – see the corresponding $A-C$ line in Figure 7. Now, if both governments avoid intervention in the liquidity market setting $\theta_A^* = \theta_B^* = 0$, $\widehat{\theta}$ will drop along the $A-B$ line to $\overline{\theta}\sqrt{2(1-R)}$, with loss of welfare. However, the governments could abolish fragmentation but still provide liquidity by pooling the maximum feasible liquidity that they can provide, T. Total liquidity will be determined at point D in Figure 5, at $\widehat{\theta} = 2T$, lower than the fragmented $\theta_A^* = \theta_B^* = T$ (at the intersection of the T constraint and the $A-B$ curve), but still above $\overline{\theta}\sqrt{2(1-R)}$, provided that T is sufficiently close to point A. Social welfare will be at the left end of an $A-C$ like curve, but lower than the $A-C$ curve (as total liquidity is lower). Provided that T is close enough to point A and by the continuity of the SW manifold, welfare is higher relative unilateral fragmentation (in spite of the loss of hot money).

The following numerical example shows that the parameter set in which full integration is optimal is non-empty: $\delta = 0.6; \rho_0 = 1.2; \underline{w} = 0.4; y = 1.25; \pi = 0.5; \overline{\theta} = 0.7$ which yields as output $b(\delta) = 0.65; \underline{q} = 0.43; (2y - \rho_1) - (1-\pi)(2y - \underline{q}) = 0.065; R = 0.501; \xi = 0.74$. ■

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